Technical note. Computing the indices

The human development index

The HDI is based on three indicators: longevity, as measured by life expectancy at birth; educational attainment, as measured by a combination of adult literacy (two-thirds weight) and the combined gross primary, secondary and tertiary enrolment ratio (one-third weight); and standard of living, as measured by real GDP per capita (PPP$).

Fixed minimum and maximum values

To construct the index, fixed minimum and maximum values have been established for each of these indicators:

- Life expectancy at birth: 25 years and 85 years.
- Adult literacy rate: 0% and 100%.
- Combined gross enrolment ratio: 0% and 100%.
- Real GDP per capita (PPP$): $100 and $40,000 (PPP$).

For any component of the HDI individual indices can be computed according to the general formula:

$$\text{Index} = \frac{\text{Actual } x_i \text{ value} - \text{minimum } x_i \text{ value}}{\text{Maximum } x_i \text{ value} - \text{minimum } x_i \text{ value}}$$

If, for example, the life expectancy at birth in a country is 65 years, the index of life expectancy for this country would be:

$$\text{Life expectancy index} = \frac{65 - 25}{85 - 25} = \frac{40}{60} = 0.667$$

Treatment of income

Constructing the income index is a little more complex. Over the years the *Human Development Report* has used a particular formula to do this, explained below. This year a thorough review of the treatment of income in the HDI was done, based on the work of Anand and Sen (1999).

Income enters into the HDI as a surrogate for all the dimensions of human development not reflected in a long and healthy life and in knowledge—in a nutshell, it is a proxy for a decent standard of living. The basic approach in the treatment of income has been driven by the fact that achieving a respectable level of human development does not require unlimited income. To reflect this, income has always been discounted in calculating the HDI. The issue is, how should it be discounted, and at what level?

In previous years the practice was to discount income above the threshold level of the world average income, using the following formula:

$$\begin{aligned} W(y) &= y^* \text{ for } 0 < y < y^* \\ &= y^* + 2[(y - y^*)^{1/2}] \text{ for } y^* < y < 2y^* \\ &= y^* + 2(y^{*1/2}) + 3[(y - 2y^*)^{1/3}] \text{ for } 2y^* < y < 3y^* \end{aligned}$$

where y is the actual per capita income in PPP$ and y^* is the threshold per capita income (PPP$) at the world average income in the year for which the HDI is constructed. The world average income was taken as the threshold income on the premise that each person should have the income that the world on average enjoys.

To calculate the discounted value of the maximum income of $40,000 (PPP$), the following formula was used:

$$W(y) = y^* + 2(y^{*1/2}) + 3(y^{*1/3}) + 4(y^{*1/4}) + 5(y^{*1/5}) + 6(y^{*1/6}) + 7[(40{,}000 - 6y^*)^{1/7}]$$

This is because $40,000 (PPP$) is between $6y^*$ and $7y^*$. With the above formula, the discounted value of the maximum income of $40,000 (PPP$) is $6,311 (PPP$).

The main problem with this formula is that it discounts the income above the threshold level very heavily, penalizing the countries in which income exceeds the threshold level. It reduces the $34,000 (PPP$) between the threshold and maximum level of income to a mere $321 (PPP$). In many cases income loses its relevance as a proxy for all dimensions of human development other than a long and healthy life and knowledge.

This year's refinement in the treatment of income attempts to rectify this problem by putting the methodology on a more solid analytical foundation. The rationale and the formula adopted in the refinement are discussed in detail in Anand and Sen (1999). To summarize, in the construction of this year's HDI, income is treated using the following formula:

$$W(y) = \frac{\log y - \log y_{\min}}{\log y_{\max} - \log y_{\min}}$$

There are several advantages to this formula. First, it does not discount income as severely as the formula used earlier (technical note figure 1). Second, it discounts all income, not just the income above a certain level. Third, as the figure shows, the asymptote starts quite late, so middle-income countries are not penalized unduly; moreover, as income rises further in these countries, they will continue to receive recognition for their increasing income as a potential means for further human development.

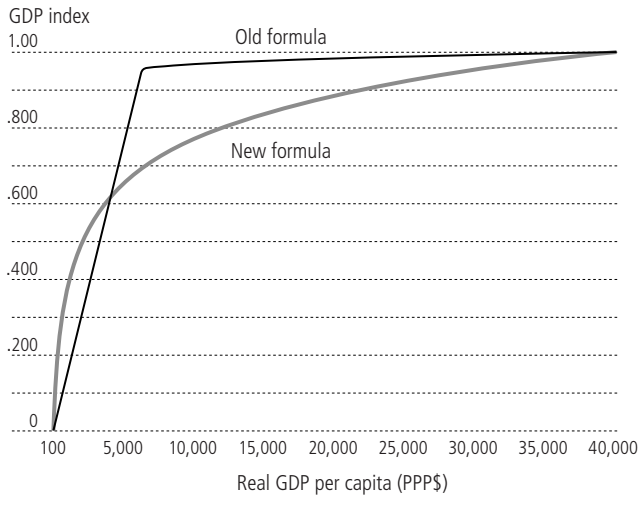
Illustration of the HDI methodology

The construction of the HDI is illustrated with two examples—Germany and China, an industrialized and a developing country.

Country	Life expectancy (years)	Adult literacy rate (%)	Combined gross enrolment ratio (%)	Real GDP per capita (PPP$)
Germany	77.2	99.0	88.1	21,260
China	69.8	82.9	68.9	3,130

FIGURE TN1

Income discounting under the old and new formulas



Source: Human Development Report Office.

Life expectancy index

$$\text{Germany} = \frac{77.2 - 25}{85 - 25} = \frac{52.2}{60} = 0.870$$

$$\text{China} = \frac{69.8 - 25}{85 - 25} = \frac{44.8}{60} = 0.747$$

Adult literacy index

$$\text{Germany} = \frac{99.0 - 0}{100 - 0} = \frac{99.0}{100} = 0.990$$

$$\text{China} = \frac{82.9 - 0}{100 - 0} = \frac{82.9}{100} = 0.829$$

Combined gross enrolment index

$$\text{Germany} = \frac{88.1 - 0}{100 - 0} = 0.881$$

$$\text{China} = \frac{68.9 - 0}{100 - 0} = 0.689$$

Educational attainment index

Germany = [2(0.990) + 1(0.881)]/3 = 0.954
China = [2(0.829) + 1(0.689)]/3 = 0.782

Adjusted real GDP per capita (PPP$) index

$$\text{Germany} = \frac{\log (21{,}260) - \log (100)}{\log (40{,}000) - \log (100)} = 0.895$$

$$\text{China} = \frac{\log (3{,}130) - \log (100)}{\log (40{,}000) - \log (100)} = 0.575$$

Human development index

The HDI is a simple average of the life expectancy index, educational attainment index and adjusted real GDP per capita (PPP$) index, and so is derived by dividing the sum of these three indices by 3.

Country	Life expectancy index	Educational attainment index	Adjusted real GDP (PPP$) index	Sum of the three indices	HDI
Germany	0.870	0.954	0.895	2.719	0.906
China	0.747	0.782	0.575	2.104	0.701

Comparing HDI values across years

The HDI values in this year's Report are not strictly comparable with those in last year's because of the change in the treatment of income in the HDI. Comparability is also affected by significant revision in the data series for some indicators, particularly the income data (PPP$) from the World Bank. As a result of these changes, both HDI values and rankings for many countries this year differ considerably from those in last year's Report. Technical note table 1 shows the changes in countries' HDI rankings and the source of those changes—whether refinement of the treatment of income or revision of data.

The table makes two clear points. First, most of the changes in HDI rankings can be attributed to the changes in the treatment of income. Second, although a simple comparison of the HDI values in last year's Report with those in this year's shows an apparent deterioration, a comparison using the new treatment of income in both cases shows an improvement in the HDI value for every country.

The gender-related development index and the gender empowerment measure

For comparisons among countries the GDI and the GEM are limited to data widely available in international data sets. For this year's Report we have endeavoured to use the most recent, reliable and internally consistent data. Collecting more extensive and more reliable gender-disaggregated data is a challenge that the international community should squarely face. We continue to publish results on the GDI and the GEM—based on the best available estimates—in the expectation that it will help increase the demand for such data.

The gender-related development index

The GDI uses the same variables as the HDI. The difference is that the GDI adjusts the average achievement of each country in life expectancy, educational attainment and income in accordance with the disparity in achievement between women and men. (For a detailed explanation of the GDI methodology see technical note 1 in *Human Development Report 1995*.) For this gender-sensitive adjustment we use a weighting formula that expresses a moderate aversion to inequality, setting the weighting parameter, ϵ, equal to 2. This is the harmonic mean of the male and female values.

The GDI also adjusts the maximum and minimum values for life expectancy, to account for the fact that women tend to live longer than men. For women the maximum value is 87.5 years and the minimum value 27.5 years; for men the corresponding values are 82.5 and 22.5 years.

Calculating the index for income is fairly complex. Values of real per capita GDP (PPP$) for women and men are calculated from the female share (s_f) and male share (s_m) of earned income. These shares, in turn, are estimated from the ratio of the female wage (w_f) to the male wage (w_m) and the percentage shares of women (ea_f) and men (ea_m) in the economically active population. When data on the wage ratio are not available, a value of 75%, the weighted mean of the wage ratios for all countries with wage data, is used. The estimates of female and male per capita income (PPP$) are treated in the same way as income is treated in the HDI and then used to compute the equally distributed income index.

$$\text{Female share of the wage bill} = \frac{(w_f/w_m) \times ea_f}{[(w_f/w_m) \times ea_f] + ea_m}$$

Assuming that the female share of earned income is exactly equal to the female share of the wage bill,

$$s_f = \frac{(w_f/w_m) \times ea_f}{[(w_f/w_m) \times ea_f] + ea_m}$$

If it is now assumed that the total GDP (PPP$) of a country ($Y$) is also divided between women and men according to s_f, the total GDP (PPP$) going to women is given by ($s_f \times Y$) and the total GDP (PPP$) to men by [$Y - (s_f \times Y)$].

Per capita GDP (PPP$) of women is $y_f = s_f \times Y/N_f$, where N_f is the total female population.
Per capita GDP (PPP$) of men is $y_m = [Y - (s_f \times Y)]/N_m$, where N_m is the total male population.

Treating income the same way as in the construction of the HDI, the adjusted income for women, $W(y_f)$, is given by:

$$W(y_f) = \frac{\log y_f - \log y_{\min}}{\log y_{\max} - \log y_{\min}}$$

The adjusted income for men, $W(y_m)$, is given by:

$$W(y_m) = \frac{\log y_m - \log y_{\min}}{\log y_{\max} - \log y_{\min}}$$

The equally distributed income index is given by:

$$\{[\text{female population share} \times (\text{adjusted female per capita PPP\$ GDP})^{-1}] + [\text{male population share} \times (\text{adjusted male per capita PPP\$ GDP})^{-1}]\}^{-1}$$

The indices for life expectancy, educational attainment and income are added together with equal weight to derive the final GDI value.

Illustration of the GDI methodology

We choose Cameroon to illustrate the steps for calculating the gender-related development index. The parameter of inequality aversion, ϵ, equals 2. (Any discrepancies in results are due to numbers' being rounded up.)

Population (millions)

Total	13.924
Females	7.009
Males	6.915

Percentage share of population

Females	50.3
Males	49.7

STEP ONE

Computing the equally distributed life expectancy index

Life expectancy at birth (years)

Females	56.0
Males	53.4

Life expectancy index

Females	$(56.0 - 27.5)/60 = 0.476$
Males	$(53.4 - 22.5)/60 = 0.516$

Equally distributed life expectancy index

$\{[\text{female population share} \times (\text{female life expectancy index})^{-1}] + [\text{male population share} \times (\text{male life expectancy index})^{-1}]\}^{-1}$

$[0.503(0.476)^{-1} + 0.497(0.516)^{-1}]^{-1} = 0.495$

STEP TWO

Computing the equally distributed educational attainment index

Adult literacy rate (percent)

Females	64.6
Males	79.0

Adult literacy index

Females	$(64.6 - 0)/100 = 0.646$
Males	$(79.0 - 0)/100 = 0.790$

Combined gross enrolment ratio (percent)

Females	39.2
Males	47.6

Combined gross enrolment index

Females	$(39.2 - 0)/100 = 0.392$
Males	$(47.6 - 0)/100 = 0.476$

Educational attainment index

2/3(adult literacy index) + 1/3(combined gross enrolment index)

Females	$2/3(0.646) + 1/3(0.392) = 0.561$
Males	$2/3(0.790) + 1/3(0.476) = 0.685$

Equally distributed educational attainment index

$\{[\text{female population share} \times (\text{educational attainment index})^{-1}] + [\text{male population share} \times (\text{educational attainment index})^{-1}]\}^{-1}$

$[0.503(0.561)^{-1} + 0.497(0.685)^{-1}]^{-1} = 0.616$

STEP THREE

Computing the equally distributed income index

Percentage share of the economically active population

Females (ea_f)	38.3
Males (ea_m)	61.7

Ratio of female non-agricultural wage to male non-agricultural wage (w_f/w_m): 0.750

GDP per capita: \$1,890 (PPP\$)

Total GDP (PPP\$): \$1,890 × 13.924 million = \$26,316 million (PPP\$)

$$s_f = \frac{0.750 \times 0.383}{(0.750 \times 0.383) + 0.617}$$

$$= \frac{0.287}{0.287 + 0.617}$$

$$= 0.318$$

Female total GDP (PPP\$) = 0.318 × \$26,316 million (PPP\$) = \$8,368 million (PPP\$)
Male total GDP (PPP\$) = \$26,316 million (PPP\$) − \$8,368 million (PPP\$)
= \$17,948 million (PPP\$)

Per capita female GDP (PPP\$) = \$8,368 million/7.009 million = \$1,194 (PPP\$)
Per capita male GDP (PPP\$) = \$17,948 million/6.915 million = \$2,596 (PPP\$)

$W(y_f) = [\log(1{,}194) - \log(100)]/[\log(40{,}000) - \log(100)]$
$= (3.076 - 2.000)/(4.602 - 2.000)$
$= 1.076/2.602$
$= 0.414$

$W(y_m) = [\log(2{,}596) - \log(100)]/[\log(40{,}000) - \log(100)]$
$= (3.415 - 2.000)/(4.602 - 2.000)$
$= 1.415/2.602$
$= 0.544$

Equally distributed income index

$(\{\text{female population share} \times [W(y_f)]^{-1}\} + \{\text{male population share} \times [W(y_m)]^{-1}\})^{-1}$

$[0.503 \times (0.414)^{-1} + 0.497 \times (0.544)^{-1}]^{-1}$
$= [0.503 \times 2.415 + 0.497 \times 1.838]^{-1}$
$= [2.128]^{-1}$
$= 0.469$

STEP FOUR

Computing the GDI

$1/3(0.495 + 0.616 + 0.469) = 0.527$

The gender empowerment measure

The GEM uses variables constructed explicitly to measure the relative empowerment of women and men in political and economic spheres of activity.

The first two variables are chosen to reflect economic participation and decision-making power: women's and men's percentage shares of administrative and managerial positions and their percentage shares of professional and technical jobs. These are broad, loosely defined occupational categories. Because the relevant population for each is different, we calculate a separate index for each and then add the two together.

The third variable, women's and men's percentage shares of parliamentary seats, is chosen to reflect political participation and decision-making power.

For all three of these variables we use the methodology of population-weighted $(1-\epsilon)$ averaging to derive an "equally distributed equivalent percentage" (EDEP) for both sexes taken together. Each variable is indexed by dividing the EDEP by 50%.

An income variable is used to reflect power over economic resources. It is calculated in the same way as for the GDI except that unadjusted rather than adjusted real GDP per capita is used.

The three indices—for economic participation and decision-making, political participation and decision-making, and power over economic resources—are added together to derive the final GEM value.

Illustration of the GEM methodology

We choose Algeria to illustrate the steps in calculating the GEM. The parameter of inequality aversion, ϵ, equals 2. (Any discrepancies in results are due to numbers' being rounded up.)

Population (millions)

Total	29.394
Females	14.518
Males	14.876

Percentage share of population

Females	49.39
Males	50.61

STEP ONE

Calculating indices for parliamentary representation and administrative and managerial, and professional and technical, positions

Percentage share of parliamentary representation

Females	3.82
Males	96.18

Percentage share of administrative and managerial positions

Females	5.9
Males	94.1

Percentage share of professional and technical positions

Females	27.6
Males	72.4

Calculating the EDEP for parliamentary representation

$[0.494(3.82)^{-1} + 0.506(96.18)^{-1}]^{-1} = 7.4$

Calculating the EDEP for administrative and managerial positions

$[0.494(5.9)^{-1} + 0.506(94.1)^{-1}]^{-1} = 11.2$

Calculating the EDEP for professional and technical positions

$[0.494(27.6)^{-1} + 0.506(72.4)^{-1}]^{-1} = 40.2$

Indexing parliamentary representation

$7.4/50 = 0.149$

Indexing administrative and managerial positions

$11.2/50 = 0.224$

Indexing professional and technical positions

$40.2/50 = 0.804$

Combining the indices for administrative and managerial, and professional and technical, positions

$(0.224 + 0.804)/2 = 0.514$

STEP TWO

Calculating the index for female and male income

Percentage share of the economically active population

Females (ea_f)	26.2
Males (ea_m)	73.8

Ratio of female non-agricultural wage to male non-agricultural wage (w_f/w_m): 0.750

Per capita GDP (PPP$): $4,460 (PPP$)

Total GDP (PPP$): $4,460 × 29.394 million = $131,097 million (PPP$)

$$s_f = \frac{0.750 \times 0.262}{(0.750 \times 0.262) + 0.738}$$

$$= \frac{0.197}{0.197 + 0.738}$$

$$= 0.210$$

Female total GDP (PPP$) = 0.210 × $131,097 million (PPP$) = $27,530 million (PPP$)
Male total GDP (PPP$) = $131,097 million (PPP$) − $27,530 million (PPP$)
= $103,567 million (PPP$)
Per capita female GDP (PPP$) = $27,530 million/14.518 million = $1,896 (PPP$)
Per capita male GDP (PPP$) = $103,567 million/14.876 million = $6,962 (PPP$)

$$\text{Index of female per capita GDP} = \frac{1{,}896 - 100}{40{,}000 - 100}$$

$$= \frac{1{,}796}{39{,}900}$$

$$= 0.045$$

$$\text{Index of male per capita GDP} = \frac{6{,}962 - 100}{40{,}000 - 100}$$

$$= \frac{6{,}862}{39{,}900}$$

$$= 0.172$$

Calculating the equally distributed income index

$[0.494(0.045)^{-1} + 0.506(0.172)^{-1}]^{-1} = 0.072$

STEP THREE

Computing the GEM

$1/3(0.149 + 0.514 + 0.072)$
$= [1/3(0.735)]$
$= 0.245$

The human poverty index

Computing the human poverty index for developing countries

The human poverty index for developing countries (HPI-1) concentrates on deprivations in three essential dimensions of human life already reflected in the HDI—longevity, knowledge and a decent standard of living. The first deprivation relates to survival—vulnerability to death at a relatively early age. The second relates to knowledge—being excluded from the world of reading and communication. The third relates to a decent living standard in terms of overall economic provisioning.

In constructing the HPI-1, the deprivation in longevity is represented by the percentage of people not expected to survive to age 40 (P_1), and the deprivation in knowledge by the percentage of adults who are illiterate (P_2). The deprivation in living standard is represented by a composite (P_3) of three variables—the percentage of people without access to safe water (P_{31}), the percentage of people without access to health services (P_{32}) and the percentage of moderately and severely underweight children under five (P_{33}).

The composite variable P_3 is constructed by taking a simple average of the three variables P_{31}, P_{32} and P_{33}. Thus

$$P_3 = \frac{(P_{31} + P_{32} + P_{33})}{3}$$

Following technical note 1 in *Human Development Report 1997,* the formula for the HPI-1 is given by:

$$\text{HPI-1} = [1/3(P_1^3 + P_2^3 + P_3^3)]^{1/3}$$

As an example, we compute the HPI-1 for Panama.

Country	P_1 (%)	P_2 (%)	P_{31} (%)	P_{32} (%)	P_{33} (%)
Panama	6.4	8.9	7.0	18.0	7.0

STEP ONE

Calculating P_3

$$P_3 = \frac{7 + 18 + 7}{3} = \frac{32}{3} = 10.7$$

STEP TWO

Constructing the HPI-1

$$\begin{aligned}\text{HPI-1} &= [1/3(6.4^3 + 8.9^3 + 10.7^3)]^{1/3}\\ &= [1/3(262.144 + 704.97 + 1{,}225.04)]^{1/3}\\ &= [1/3(2{,}192.15)]^{1/3}\\ &= 730.72^{1/3}\\ &= 9.0\end{aligned}$$

Computing the human poverty index for industrialized countries

The human poverty index for industrialized countries (HPI-2) concentrates on deprivations in four dimensions of human life, quite similar to those reflected in the HDI—longevity, knowledge, a decent standard of living and social exclusion. The first deprivation relates to survival—vulnerability to death at a relatively early age. The second relates to knowledge—being deprived of the world of reading and communication. The third relates to a decent standard of living in terms of overall economic provisioning. And the fourth relates to non-participation or exclusion.

In constructing the HPI-2, the deprivation in longevity is represented by the percentage of people not expected to survive to age 60 (P_1), and the deprivation in knowledge by the percentage of people who are functionally illiterate as defined by the OECD (P_2). The deprivation in standard of living is represented by the percentage of people living below the income poverty line, set at 50% of the median disposable personal income (P_3). And the fourth deprivation, in non-participation or exclusion, is measured by the rate of long-term (12 months or more) unemployment (P_4) of the labour force.

Following technical note 1 in *Human Development Report 1997,* the formula for the HPI-2 is given by:

$$\text{HPI-2} = [1/4(P_1^3 + P_2^3 + P_3^3 + P_4^3)]^{1/3}$$

As an example, we compute the HPI-2 for the United States.

Country	P_1 (%)	P_2 (%)	P_3 (%)	P_4 (%)
United States	12.6	20.7	19.1	0.5

Constructing the HPI-2

$$\begin{aligned}\text{HPI-2} &= [1/4(12.6^3 + 20.7^3 + 19.1^3 + 0.5^3)]^{1/3}\\ &= [1/4(2{,}000.4 + 8{,}869.7 + 6{,}967.9 + 0.125)]^{1/3}\\ &= [1/4(17{,}838.1)]^{1/3}\\ &= 4{,}459.5^{1/3}\\ &= 16.5\end{aligned}$$