Technical notes

1. Computing gender-equity-sensitive indicators

Over the past five years, a great deal has been achieved by the *Human Development Report* in shifting the focus of attention of the world community from such mechanical indicators of economic progress as GNP and GDP to indicators that come closer to reflecting the well-being and opportunities actually enjoyed by populations. Although the *Human Development Report* has been widely read primarily because of the extensive and detailed statistical analyses of achievements and limitations in the living conditions of people in different parts of the contemporary world, the aggregative human development index (HDI) also has played some part in bringing about this reorientation. Despite the obvious limitations of the HDI (arising in part from its attempt to capture a complex reality in a summary form with imperfect data), it has served as something of a rival to the other summary indicator—the aggregative GNP, which hitherto had been almost universally used as the premier index of the economic achievement of nations. The HDI clearly has been able to present some aspects of human development that the GNP tends to miss.

From the beginning, the *Human Development Report* has been concerned with inequalities in the opportunities and predicaments of women and men. Although this perspective has received some attention in past Reports, there is a strong case at this time for concentrating specifically on that issue for a more comprehensive investigation of gender inequality in economic and social arrangements in the contemporary world.

In performing this task, there is need for fresh economic and social analyses as well as careful and probing empirical research. Women and men share many aspects of living together, collaborate with each other in complex and ubiquitous ways, and yet end up—often enough—with very different rewards and deprivations. This note is specifically concerned with developing a framework for "gender-equity-sensitive indicators" of achievements and freedoms. The methodology for this is explored in the sections that follow.

Group inequality and aggregation: the basic structure

We may begin by examining the inequality between women and men in a dimension in which the "potentials" of the two groups do not differ. Literacy is an obvious example. In contrast, in the case of life expectancy, we must take note of the evident biological advantage in survival of females over males (see Waldron 1983, Sen 1992b, Anand 1993 and the references cited there). Given symmetric treatment in nutrition, health care and other conditions of living (including the duration and intensity of work), women have systematically lower age-specific mortality rates than men, resulting in a life expectancy for women that is significantly higher than that for men—possibly by some five years or more. There is no corresponding difference in the potential for adult literacy (that is, in the percentage of the population aged 15 and above that is literate).

The assessment of relative inequality in achievement can be reasonably clear when there are only two groups—as in the case of gender. The larger the gender gap, holding the overall mean constant, the larger is inequality as measured by any index in the Lorenz class (see Anand 1983, appendix D); this class includes most commonly used inequality measures, such as the Gini coefficient, the two Theil indices, the Atkinson index and the squared coefficient of variation. A bigger gender gap, with the same overall mean (and the same population proportions of the two groups), is equivalent to a simple *mean-preserving regressive transfer*. (In terms of Lorenz curves, this would correspond to an unambiguously lower curve.) In the special two-group case, disparity ratios *or* gaps will clearly reflect the inequality in achievement between the two groups. Given equality preference and the same overall mean, more relative inequality will indicate a worse social state of affairs, and this evaluative feature must be reflected in the gender-equity-sensitive indicators.

This simple recognition still leaves open the question of what would be appropriate standards of comparison when the overall or mean levels of achievement are different. In particular, how might we think about "trading off" more relative equality against a higher absolute achievement? Haiti, for example, has a total literacy rate of 43%—40% for females and 46% for males. Should this social outcome be judged worse or better than that of Chad, which has a total literacy rate of 45%, with a rate

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of 31% for females and 59% for males? Haiti has less gender inequality in literacy than Chad, but it also has a lower overall rate of literacy. A comparison between the two countries now calls for some way of assessing the comparative claims of more relative equality against higher absolute achievement. An explicit evaluative exercise on this "trade-off" is required.

We begin with the approach explored by A. B. Atkinson (1970) for measuring relative income inequality and extend this analysis to fit our task (see also Kolm 1969, Sen 1973, Osmani 1982, Anand 1983 and Blackorby and Donaldson 1984). Let X be the indicator of achievement, and let X_f and X_m refer to the corresponding female and male achievements. If n_f and n_m are the numbers of females and males in the population, the overall or mean achievement $\bar{X}$ is given by

$$\bar{X} = (n_f X_f + n_m X_m)/(n_f + n_m).$$

We posit a social valuation function for achievement that is additively separable, symmetric and of constant elasticity marginal valuation form

$$V(X) = \begin{cases} \frac{1}{1-\epsilon} X^{1-\epsilon} & \epsilon \geq 0,\ \epsilon \neq 1 \\ \log X & \epsilon = 1 \end{cases}$$

up to a positive affine transformation. Only values of $\epsilon \geq 0$ are considered so as to reflect a preference for equality in the social valuation function.

For any pair (X_f, X_m) of female and male achievements, we can construct an "equally distributed equivalent achievement" X_{ede}. This is defined to be the level of achievement that, if attained equally by women and men, as (X_{ede}, X_{ede}), would be judged to be exactly as valuable socially as the actually observed achievements (X_f, X_m). According to the formula for social valuation, for a given ϵ, X_{ede} is thus defined through the equation

$$(n_f + n_m)\frac{X_{ede}^{1-\epsilon}}{1-\epsilon} = n_f \frac{X_f^{1-\epsilon}}{1-\epsilon} + n_m \frac{X_m^{1-\epsilon}}{1-\epsilon},$$

which implies that

$$X_{ede} = (n_f X_f^{1-\epsilon} + n_m X_m^{1-\epsilon})^{\frac{1}{1-\epsilon}} / (n_f + n_m)^{\frac{1}{1-\epsilon}}$$
$$= (p_f X_f^{1-\epsilon} + p_m X_m^{1-\epsilon})^{\frac{1}{1-\epsilon}},$$

where we define the proportions $p_f = n_f/(n_f + n_m)$ and $p_m = n_m/(n_f + n_m)$. Hence, X_{ede} is formed from (X_f, X_m) by taking what we shall call a "$(1-\epsilon)$ average" of X_f and X_m rather than a simple arithmetic average of the female and male achievements.[1] When $\epsilon = 0$, X_{ede} reduces to $\bar{X}$, the simple arithmetic average; here there is no concern for equality, and the arithmetic mean indicates the social achievement. But when $\epsilon > 0$, there is a social preference for equality (or an aversion to inequality) that is measured by the magnitude of the parameter ϵ.

Assuming that female achievement falls short of male achievement—that is, $(0 \leq)\ X_f < X_m$—the following results can be demonstrated for $(1-\epsilon)$ averaging:

1. $X_f \leq X_{ede} \leq X_m$.
2. The larger ϵ is, the smaller is X_{ede} (given $X_f, X_m > 0$).
3. $X_{ede} \leq \bar{X}$ for $\epsilon \geq 0$ (with equality holding when $\epsilon = 0$).
4. $X_{ede} \to X_f$ as $\epsilon \to \infty$.

Result 4 corresponds to the Rawlsian maximin situation in which social achievement is judged purely by the achievement of the worst-off group, which in the case of gender typically refers to women.[2] If $X_f < X_m$ in every country and if $\epsilon \to \infty$ (equity preference tending to infinity), social achievement across countries will be measured by female achievement alone: in the averaging, the weight given to male achievement in excess of female achievement will tend to zero. In this case, the equally distributed equivalent achievement index X_{ede} reduces to the index for the relatively deprived group (typically women), and countries are ranked according to the absolute achievement of women in those countries.

As mentioned earlier, X_{ede} is a $(1-\epsilon)$ average of X_f and X_m. When $\epsilon = 0$, $X_{ede} = \bar{X}$, the *arithmetic* average of X_f and X_m. When $\epsilon = 1$, X_{ede} is the *geometric* average; and when $\epsilon = 2$, X_{ede} is the *harmonic* mean of X_f and X_m.[3] When $\epsilon \to \infty$, $X_{ede} \to \min\{X_f, X_m\}$. The equally distributed equivalent achievement X_{ede} can be calculated for each country for different values of ϵ, the parameter of equity preference. Thus, if the preference for equity is very small (ϵ close to 0), Chad's literacy rate of 31% for females and 59% for males, corresponding to an overall literacy rate of 45%, will be judged to be better than Haiti's rate of 40% for females and 46% for males, corresponding to an overall rate of 43%. As the equity preference parameter ϵ is raised, Haiti's achievement will overtake that of Chad's; in the limit, as ϵ tends to infinity, Haiti's equally distributed equivalent achievement will be 40% and Chad's 31%. For all values of ϵ above the critical cut-off of 1.2, at which the two countries' achievements are the same, Haiti's achievement will be judged to be better than Chad's.

The equally distributed equivalent achievement X_{ede}, applied to gender differences, yields a measure that is, in fact, a gender-equity-sensitive indicator (GESI). This is, of course, an index of overall achievement taking note of inequality, rather than a measure of gender equality. But it uses—explicitly or by implication—equity-sensitive weights on the achievements of the two groups, rather than the unweighted mean of the two sets of achievements that is more commonly used (including, hitherto, in the *Human Development Report*). It incorporates implicitly something like a gender equality index. The index of relative equality E that underlies X_{ede} can be defined simply as

$$E = X_{ede}/\bar{X}.$$

This can vary from 0 to 1 as equality is increased.[4] Hence, the measure of social achievement $X_{ede} = E \cdot \bar{X}$ is simply the relative equality index E multiplied by the overall or mean achievement measure $\bar{X}$. Relative equality and mean absolute achievement are thus integrated into the gender-equity-sensitive indicators.

Equity-sensitive aggregation and life expectancy

So far, the analysis has been confined to achievements in which the "potentials" of women and men do not differ (for example, each group has the same range of achievable literacy, from 0% to 100%). The situation is different, however, when it comes to mortality rates and life expectancy, as was mentioned earlier. Given the evidence of biological differences in survival rates favouring women (with comparable care), we are forced to address the question of the appropriate comparable scales of achievement in life expectancy for women and men. And we have to integrate that differential scaling into the general evaluative scheme of gender-equity-sensitive indices.

There is indeed strong evidence that the maximum potential life expectancy for women is greater than that for men—given similar care, including health care and nutritional opportunities (see Holden 1987, Waldron 1983 and the references cited there). Indeed, in most industrial countries, women typically outlive men by six to eight years. Women's higher potential life expectancy is anticipated in demographic projections as well. For the year 2050, for example, life expectancy in industrial countries is projected at 87.5 years for women and 82.5 years for men, averaging to 85 years (see UNDP 1993c).

In considering the disaggregation of the human development index by gender, the *Human Development Report* has used separate goal posts for maximum life expectancy for females and males of 87.5 years and 82.5 years, reflecting a five-year gender gap. Minimum life expectancy has been taken to be 27.5 years for women and 22.5 years for men, giving the same range of variation (60 years) for both sexes. When no adjustment is made for gender inequality, a unit increase in longevity for either sex will contribute the same increment to the overall HDI.

In the disaggregation of the HDI in the *Human Development Report*, female and male achievements in life expectancy, X_f and X_m, have been assessed through

$$X_f = (L_f - 27.5)/60,$$
$$\text{and } X_m = (L_m - 22.5)/60.$$

The simple arithmetic average $\bar{X}$ of X_f and X_m, assuming female and male population shares of one-half each, is then calculated as

$$\bar{X} = \frac{1}{2}X_m + \frac{1}{2}X_f$$
$$= (\bar{L} - 25)/60,$$

where $\bar{L} = (L_f + L_m)/2$ is the average life expectancy attained in the population.

Equality between persons can be defined in two quite distinct ways: in terms of *attainments*, or in terms of the *shortfalls* from the maximum values that each can attain. For "attainment equality" of achievement, we have to compare the absolute levels of achievement. For "shortfall equality", what must be compared are the shortfalls of actual achievement from the maximum achievements of each group. Each of the two approaches has considerable interest (see Sen 1992a, chapter 6). Shortfall equality takes us in the direction of equal use (relative or absolute) of the respective potentials. In contrast, attainment equality is concerned with equal absolute levels of achievement (irrespective of the maximum potentials).

In those cases in which human diversity is so powerful that it is impossible to equalize the maximum levels that are potentially achievable, there is a basic ambiguity in assessing achievement and in judging equality of achievement (or of the freedom to achieve). If the maximum achievement of person 1—under the most favourable circumstances—is, say, x, and that for person 2 is $2x$, equality of attainment would invariably leave person 2 below her potential achievement. Partly as a response to such issues, Aristotle incorporated, in his *Politics*, a parametric consideration of what a person's "circumstances admit" and saw his "distributive conception" in that light. "For it is appropriate, if people are governed best that they should do best, *in so far as their circumstances admit*—unless something catastrophic happens."[5] It is possible to question this Aristotelian view in terms of the more rough-and-ready rationale of attainment equality, but there is force in the conception of shortfall equality as well, and it is that approach that is used here for assessing gender equality in the context of life expectancy variations. The gender-equity-sensitive indicators can also be made to take note of the logic behind this approach.

Thus, the approach to adjusting for gender inequality in achievement in the case of life expectancy must first involve a rescaling to take note of the potentially greater longevity of women. Such adjustments are, in fact, a part of the methodology already used for the *Human Development Report*, since these rescalings must be done whether or not we wish to take explicit note of gender inequality. But rather than taking a simple arithmetic average $\bar{X}$ of the female and male achievements X_f and X_m, we take a $(1 - \epsilon)$ average with $\epsilon > 0$. As before, we form the average X_{ede}, given for $\epsilon \neq 1$ through

$$X_{ede}^{1-\epsilon} = \frac{1}{2}X_f^{1-\epsilon} + \frac{1}{2}X_m^{1-\epsilon},$$

which reduces to $\bar{X}$ when $\epsilon = 0$.[6] Thus, we define L_{ede} through

$$[(L_{ede} - 25)/60]^{1-\epsilon} = \frac{1}{2}[(L_f - 27.5)/60]^{1-\epsilon}$$
$$+ \frac{1}{2}[(L_m - 22.5)/60]^{1-\epsilon}.$$

When $\epsilon = 0$, $L_{ede} = \bar{L}$. For $\epsilon > 0$, $L_{ede} < \bar{L}$.

Gender differences in earnings and rewarded employment

The human development index for a country consists of the average of three components: life expectancy, educational attainment and adjusted per capita income. For the gender-equity-sensitive HDI, called the gender-related development index (GDI), we simply replace the arithmetic average attainments in each component by the equally distributed equivalent achievements. Thus, the first component, $(\bar{L} - 25)/60$, is replaced by $(L_{ede} - 25)/60$. Similarly, educational attainment is replaced by the equally distributed equivalent achievement of the educational rates for females and males. No corresponding correction can be made for the third component of the HDI, because gender-specific attributions of income per head cannot be readily linked to the aggregate GDP per capita used in these calculations, and inequalities within the household are difficult to characterize and assess.

It is important to distinguish between two different aspects of income: *earnings* and *use*. If we wish to concentrate on the *use* aspect, the within-family division of income use between women and men would have to be identified to assess income use by gender. But the empirical and conceptual problems in getting at these divisions within the family are formidable indeed.

In contrast, the earnings aspect looks at women and men not as income users, but as people who *earn* incomes. Total gross national product can then be seen in terms of aggregate earnings of all women and all men, making up something like the total national income. An approximate idea of the income earnings of women and men can be obtained by looking at their employment ratios and at their relative wages.

What significance can be attached to such income earning estimates? Indeed, there is some tension in concentrating on the earnings aspect when the entire approach of the *Human Development Report* has been based on what people get out of the means they can use, rather than on the means they earn—possibly to be used by their families. But the earnings contrasts between men and women do point to an important asymmetry in nearly all societies. While women very often work as hard as—or harder than—men, much of their work is unpaid (see, for example, Goldschmidt-Clermont 1982 and 1993, Folbre 1991 and 1994, Urdaneta-Ferrán 1993 and the references cited there). There is also considerable evidence indicating that earning explicitly recognized "incomes" and working in sectors that are treated as evidence of being "economically active" can significantly and favourably influence the "deal" that women tend to get in the division of benefits and chores in the family.[7]

There is thus a case for gender division even for the real income component of the HDI, to try to note the differences between the earnings of women and men. It would be hard to get anything like the degree of precision for earnings "allocated" between men and women on the basis of rough calculations that gender-specific measures of literacy or life expectancy can offer. But even estimates of the relative earnings of women and men would give the gender-equity-sensitive indicator another component with some bite. For such estimates, the total GDP per capita could be notionally "split" between women and men in the ratio of the products of their employment rates and wage rates per unit of employment. It would then be necessary, however, to explain clearly that (1) this procedure looks at income from the earnings perspective rather than the use perspective (even though gender inequalities seem to link use to earnings), and (2) the evaluations of earnings of women and men are fairly "soft" estimates, which should be interpreted with much caution.

Extent of inequality aversion ϵ

As discussed earlier, the values of the parameter ϵ can be taken to range from zero to infinity, reflecting the extent of social preference for equality. As a parameter, ϵ stands for the elasticity of the marginal social valuation of achievement, and tells us how quickly the marginal value falls as achievement rises (that is, how strongly diminishing the marginal social returns are). ϵ can, in fact, be seen as a reflection of the extent of inequality aversion. When ϵ is taken to be zero, there is no decline in marginal values, so the simple arithmetic mean does well enough. At the other extreme, when ϵ is taken to be infinity, the sensitivity is so great that we end up picking only the lower of the two numbers in a pair, ignoring the achievement of the better-off. It would be interesting to calculate the GDI, the gender-equity-sensitive adaptation of the HDI, for several parametric values of ϵ, such as 0, 1, 2, 3, 5, 10 and ∞. We typically will use the value $\epsilon = 2$.

The implications of different choices of ϵ can be gauged by examining the effects on X_{ede}, the equally distributed equivalent achievement. We can compare the relative increase in X_{ede} through a unit increase in female achievement X_f compared with a unit increase in male achievement X_m. From Anand and Sen (1995, appendix A.1, equation 2), we have

$$\frac{\partial X_{ede} / \partial X_f}{\partial X_{ede} / \partial X_m} = \frac{p_f V'(X_f)/V'(X_{ede})}{p_m V'(X_m)/V'(X_{ede})}$$

$$= \frac{V'(X_f)}{V'(X_m)} \quad \text{assuming } p_f = p_m = \frac{1}{2}$$

$$= X_f^{-\epsilon} / X_m^{-\epsilon} = (X_m / X_f)^{\epsilon}$$

if the social valuation function $V(X)$ has a constant elasticity of marginal valuation ϵ.

According to this, if male achievement X_m is *twice* female achievement X_f—that is, $X_m/X_f = 2$—and if $\epsilon = 1$ (that is, we have the logarithmic form for $V(X)$), a unit increase in female achievement would contribute twice as much to X_{ede} as a unit increase in male achievement (see technical note table 1.1). If X_m/X_f remains equal to 2, but $\epsilon = 2$, a unit increase in female achievement contributes four times as much as a unit increase in male achievement. Holding X_m/X_f constant (at any value above 1), as ϵ is increased there is an increase in the relative contribution to X_{ede} from a unit increase in X_f compared with a unit increase in X_m. Technical note table 1.1 estimates the relative contribution to X_{ede} of a unit increase in female achievement compared with a unit increase in male achievement for different values of ϵ and for different ratios of male achievement to female achievement (X_m/X_f).

How much would the GDI differ from the HDI (bearing in mind that the HDI is, in fact, a special case of the GDI, with $\epsilon = 0$)? Clearly, the distributional correction would tend to pull the value of HDI down, and we expect the GDI systematically to be significantly less than the corresponding HDI values, for relatively high values of ϵ.

Relative gender equality can be reasonably captured by comparing the values of the gender-equity-sensitive indicator with the uncorrected average measure. That average (gender-blind) measure is based on taking an arithmetic average (as with the HDI) over the entire population, whereas the formula for the GESI permits an

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Relative contributions to X_{ede} of unit increases in X_f and X_m for alternative values of ϵ and X_m/X_f

	Value of ϵ						
X_m/X_f	0.0	1.0	2.0	3.0	5.0	10.0	∞
1.0	1	1.0	1.0	1.0	1.0	1.0	1
1.5	1	1.5	2.3	3.4	7.6	57.7	∞
2.0	1	2.0	4.0	8.0	32.0	1,024.0	∞
2.5	1	2.5	6.3	15.6	97.7	9,536.7	∞
3.0	1	3.0	9.0	27.0	243.0	59,049.0	∞
4.0	1	4.0	16.0	64.0	1,024.0	1,048,576.0	∞

Note: The relative contributions to X_{ede} in this table, that is, the values of $(\partial X_{ede}/\partial X_f)/(\partial X_{ede}/\partial X_m)$, are estimated under the assumptions that $p_f = p_m = 1/2$ and that $V(X)$ has a constant elasticity of marginal valuation ϵ.

entire class of $(1-\epsilon)$ averaging to take note of—and to weigh against—inequalities. In the special case in which ϵ is taken to be 2, the formulae of the GESI and the GDI correspond to the harmonic mean. The equally distributed equivalent achievement corresponding to $\epsilon = 2$, that is, $X_{ede}(2)$, is then given (for equal proportions of women and men) by the formula

$$X_{ede}(2)^{-1} = \frac{1}{2}X_f^{-1} + \frac{1}{2}X_m^{-1}.$$

Hence,

$$X_{ede}(2) = 2[(1/X_f) + (1/X_m)]^{-1},$$

which is the harmonic mean of X_f and X_m. If we take the ratio of the harmonic mean to the arithmetic mean, we then get a measure of gender equity that has obvious interest.

It must be remembered that the GESI formula can also be applied to other variables chosen to represent differences in gender achievements. We must, in general, distinguish between (1) the GESI formula of $(1-\epsilon)$ averaging, and (2) the "space" on which it is applied (that is, the variables for which achievements and gender disparities are scrutinized). Even though the argument in this technical note has been developed in terms of the "classic" components of human development indicators, the GESI formula can be applied generically to any gender disparity.

Notes

1. Considering X_{ede} as a function of ϵ, we can write

$$X_{ede}(\epsilon) = (p_f X_f^{1-\epsilon} + p_m X_m^{1-\epsilon})^{\frac{1}{1-\epsilon}}.$$

For $X_f, X_m > 0$, $X_{ede}(\epsilon)$ is well defined for all ϵ (positive or negative) except $\epsilon = 1$. As $\epsilon \to 1$, we can show that $\log X_{ede}(\epsilon) \to (p_f \log X_f + p_m \log X_m)$, that is, the logarithm of the geometric mean of X_f and X_m; hence, $X_{ede}(\epsilon)$ tends to the geometric mean of (X_f, X_m). If one of the X_i, say X_f, is equal to 0, then $X_{ede}(\epsilon)$ is well defined for $\epsilon < 1$. But for $\epsilon > 1$, $X_f^{1-\epsilon} = 1/X_f^{\epsilon-1} \to \infty$ as $X_f \to 0$. In this case,

$$X_{ede}(\epsilon) = 1/[(p_f/X_f^{\epsilon-1}) + (p_m/X_m^{\epsilon-1})]^{\frac{1}{\epsilon-1}},$$

so that $p_f/X_f^{\epsilon-1}$ and the entire denominator of $X_{ede}(\epsilon)$ tends to infinity as $X_f \to 0$. Therefore, for $\epsilon > 1$, $X_{ede}(\epsilon) \to 0$ as $X_f \to 0$. Putting together the cases $\epsilon = 1$ and $\epsilon > 1$, the limiting value of $X_{ede}(\epsilon)$ for $\epsilon \geq 1$ is zero, as one of the X_i, for example, X_f, tends to zero. Thus, we may simply *define* $X_{ede}(\epsilon) = 0$ for $\epsilon \geq 1$ when X_f or X_m is equal to zero.

2. There is some ambiguity about whether this "extreme inequality aversion" leads to simple maximin, or to the lexicographic version of maximin (sometimes called "leximin"), on which see Hammond 1975.

3. By result 2 above, we have the following relationship between the three means when the two numbers X_f and X_m are positive and different: the harmonic mean is less than the geometric mean, and the geometric mean is less than the arithmetic mean.

4. The corresponding measure of relative inequality I is simply the Atkinson index:

$$I = 1 - (X_{ede}/\bar{X}).$$

Under the assumptions made on $V(X)$ in this note, both E and I are mean-independent measures. Indeed, the constant elasticity marginal valuation form is both sufficient and necessary for E and I to be homogeneous of degree zero in (X_f, X_m).

5. The translation is from Nussbaum (1988), who also discusses the precise role that this qualification plays in Aristotle's "distributive conception" (pp. 146–50; italics added).

6. On the other hand, for $\epsilon = 1$, X_{ede} is given through the logarithmic functional form. These formulations are based on the presumption that there are the same number of women as of men—hence the half-and-half division. When this does not hold, the gross mean and the gender-equity-sensitive measure involve weighting the achievements of each group by their population shares, p_f and p_m.

7. For references to the literature on this, and an analysis of why this relationship is observed in situations of "cooperative conflict" (as family living typically is), see Sen 1990a.

2. Computing the GDI and the GEM

The gender-related development index

The gender-related development index (GDI) uses the same variables as the HDI. The difference is that the GDI adjusts the average achievement of each country in life expectancy, educational attainment and income in accordance with the degree of disparity in achievement between women and men.

For this gender-sensitive adjustment, we use a weighting formula that expresses a moderate aversion to inequality, setting the weighting parameter, $\in$, equal to 2. This is the harmonic mean of the male and female values.

The harmonic mean is calculated by taking the reciprocal of the population-weighted arithmetic mean of the female and male achievement levels (which are themselves expressed in reciprocal form). Although this may sound complicated, the basic principle is straightforward. The harmonic mean will be less than the arithmetic mean to the degree that there is disparity between female and male achievement.

The first step in the calculation of the GDI is to index the variables for life expectancy and educational attainment. Although the range for life expectancy is the same for women and men (60 years), the maximum and minimum values are different. The maximum value (or "fixed goal post") for male life expectancy is 82.5 years and the minimum value is 22.5 years. For female life expectancy, the maximum value is 87.5 years and the minimum 27.5 years. The values for women and men are indexed accordingly.

The variable for educational attainment is a composite index. It includes adult literacy, with a two-thirds weight, and gross combined primary, secondary and tertiary enrolment, with a one-third weight. Each of these subcomponents is indexed separately. Both indices use a maximum value of 100% and a minimum value of 0%. The two indices are added together with the appropriate weights to form the composite index for educational attainment.

The income variable

The calculation of the index for income is more involved. In calculating the female and male shares of earned income, we use two pieces of information: the ratio of the average female wage to the average male wage and the female and male percentage shares of the economically active population aged 15 and above.

The ratio of the average female wage to the average male wage is available for the non-agricultural sector for 55 countries. This ratio is assumed to be the average ratio for the agricultural sector as well. The average ratio of female to male wages (75%) derived for these 55 countries is then applied to the countries among the 130 for which ILO sources lack such data. In fact, the wage ratio is slightly higher for the 24 industrial countries (76.2%) and slightly lower for the 31 developing countries (73%). In view of this small difference, we use the 75% ratio for all countries without data.

This ratio is a crude proxy for gender income differentials in paid work. Some countries have relatively low ratios of female to male wages because, for example, unlike many other countries, they collect data on part-time work. The wage data for gender comparisons need to be considerably improved, but failing to include this variable in our analysis would lead to women's estimated earned income share being grossly overstated.

We consider our estimates of disparity in earned income between women and men to be conservative. The 75% wage ratio is likely to be an underestimate of actual income differentials between women and men, because it does not take into account, for example, income disparities based on non-labour resources, such as land or physical capital. Since men own most property, the disparity between women and men in non-labour income would tend to be greater than that in labour income.

The second step in calculating gender disparity in income uses available information on the percentage share of men and women in the economically active population aged 15 and above. Because of the lack of data on employment by gender, this procedure makes the simplifying assumption that female employment and male employment are proportional to female and male participation in the labour force. From the ratio of female to male wages we can derive two ratios: the ratio of the female wage to the overall average wage and the ratio of the male wage to the overall average wage.

These two ratios are derived from the following definition of the total wage bill (WL):

$$WL = W_f L_f + W_m L_m,$$

where W is the average wage and L the total labour force, and the f subscript denotes female, the m subscript male.

Dividing this equation through by $W_m L$, we can solve for W/W_m:

$$W/W_m = (W_f/W_m)(L_f/L) + (W_m/W_m)(L_m/L).$$

We take the reciprocal of this result to solve for W_m/W. We can now also solve for W_f/W:

$$W_f/W = (W_f/W_m)/(W/W_m).$$

A rough estimate of the female share of income can then be derived by multiplying the ratio of the average female wage to the overall average wage by the female share of the economically active population. The male share of income can be calculated in the same way or by subtracting the female share from 1.

The third step in estimating gender disparities in income is to calculate the female and male shares of income as proportions of the female and male shares of the population. The average adjusted real GDP per capita is then discounted on the basis of the gender disparity in proportional income shares. In using adjusted real GDP per capita, we are already taking into account the diminishing marginal importance for human development of additional income above the average world per capita income. Up to this point, the methodology is the same as that used for the human development index.

The discounting for gender disparity is calculated as follows. We form two proportional income shares by dividing the female and male shares of income by the female and male shares of the population. If there were gender equality, each proportional share would be equal to 1. We then apply the GESI methodology of $(1 - \epsilon)$ averaging—with ϵ equal to 2 in this case—to the two proportional income shares to derive the "equally distributed proportional income share". The more gender inequality there is, the lower this ratio will be relative to 1. We then multiply the average real adjusted GDP per capita by the equally distributed proportional income share to derive a measure of GDP per capita that, in effect, is now discounted for gender inequality. If there were no gender inequality, the ratio would be equal to 1 and GDP per capita would remain the same. As in the HDI, real adjusted GDP per capita is the proxy for access to the basic resources necessary for human development. Finally, we index the discounted value of GDP per capita with respect to the maximum of $5,448 and the minimum of $100. These values are the same as those used in the HDI.

The last step in calculating the GDI is to add the index for income that we have just derived to the indices for life expectancy and educational attainment and divide by 3. That gives each index a one-third weight.

Note on income

Income can be seen in two ways: (1) as a resource for the *use* of the family to meet its needs and wants, and (2) as *earnings* by individuals that may or may not be aggregated for use by a united family. The "use" measure is hard to disaggregate because the family's resources are shared in ways that we cannot directly observe. But the "earnings" measures are, in principle, separable because different members of the family would tend to have separately earned incomes. Although we have tried to estimate these earnings figures for women and men, it must be noted that they need not reflect the use that women and men can make of these resources because the resources are pooled for joint use by the family.

The way that income and other resources are shared among members of a household—the intrahousehold distribution of resources—is an important factor in determining the well-being of women. This distribution varies from society to society because it is an important part of the division of labour and responsibilities in society between women and men.

These sociological patterns have been documented in many studies, but because the information is not always quantified or complete, reliable data are not available on women's access to resources for consumption. The income variable in the measures highlighted in this Report (the GDI and the GEM) therefore does not reflect women's access to income for consumption or for other uses. Instead, it indicates their capacity to *earn* income, which is a reflection of their economic independence.

Note on aggregation

The procedure used for inequality correction—in the GDI and the GEM—involves estimating inequality-corrected achievements in terms of different focus variables, and then putting them together in one aggregate measure of inequality-adjusted performance. In some respects, this procedure is a little deceptive, because the different variables might, in principle, work in somewhat opposite directions, moderating the influence of one another on the inequality among individuals. For example, if person A has a higher achievement in longevity but person B does better in education, it could be thought that these inequalities must counteract each other to some extent, so that A and B would be less unequal in a weighted aggregate of achievements than in each of the variables. And this case would differ from one in which one of the individuals is better off in both variables. In the procedure used here, we cannot discriminate between these two types of cases, since the aggregation is done by first using specific variables and then putting them together in an index of overall achievement.

This defect is inescapable at the individual level, however, given the data availability. There is no obvious way of relating individual identities in the distribution of one variable with those in the distribution of another. There is thus no serious alternative to the kind of procedure we have used. But this is not an important limitation in this context, in part because deprivations often go together and reinforce—rather than counteract—each other. For example, an educationally deprived person often is also the one with shorter longevity, as we know from statistical studies of development characteristics.

More important, it should be borne in mind that the adjustment for gender equity is being done here at a high level of aggregation, dealing with the mean positions of women and men. At this aggregated level, the inequalities almost always go together, with women in a more deprived position, on average, than men. The exceptions come in a handful of countries—such as the Nordic countries—where in one variable, life expectancy, men seem to have fallen behind women, even after the standard differences are corrected for (with five extra years expected in female life expectancy). In such cases, the inequality in life expectancy may go in a direction opposite to the inequality in education or income earning. If note were to be taken of this connection, these countries would be placed higher in overall achievement, because the inequality adjustments would have counteracted one another to some extent. But because these countries are in any case near or at the top of the international "league tables", the effect of this correction would be only to reinforce that positional lead.

Note on the GDI over time and comparing the GDI and the HDI

In calculating the GDI from 1970 to 1992 for 79 countries, we used a minimum of $0 for the income variable and, for 1970, an average ratio of female to male wages of 71% for countries that did not provide such data. The 71% ratio was the average for countries that reported wages by gender.

The GDI for 1992 is calculated directly from the values of female and male income, educational attainment and life expectancy. The HDI for 1992 is calculated directly from the average value of each indicator. If GDI values were estimated on the basis of $\epsilon = 0$, the GDI and the HDI should be equal, but minor discrepancies can arise because the female and male population weights used for the national average do not always correspond to the population weights for the female and male values that are reported separately.

For life expectancy, the GDI uses a minimum of 27.5 years for women and 22.5 years for men. That is done to maintain consistency with the HDI, which uses a minimum of 25 years for the population as a whole. In the future, we will re-examine this minimum and the five-year differential between women and men.

Illustration of the GDI methodology

We choose Paraguay to illustrate the methodology of the gender-related development index. The values for the variables used in our calculations are as follows:

Life expectancy
Females 71.9 years
Males 68.1 years
Adult literacy
Females 89.50%
Males 92.91%
Primary, secondary and tertiary enrolment
Females 58.0%
Males 59.3%

STEP ONE
Computing indices for life expectancy and education

Life expectancy
Females $(71.9 - 27.5)/60 = 0.740$
Males $(68.1 - 22.5)/60 = 0.760$
Adult literacy
Females $(89.50 - 0)/(100 - 0) = 0.895$
Males $(92.91 - 0)/(100 - 0) = 0.929$
Primary, secondary and tertiary enrolment
Females $(58.0 - 0)/(100 - 0) = 0.580$
Males $(59.3 - 0)/(100 - 0) = 0.593$
Educational attainment
Females $1/3(0.580) + 2/3(0.895) = 0.790$
Males $1/3(0.593) + 2/3(0.929) = 0.817$

STEP TWO
Computing proportional income shares

Percentage share of the economically active population
Females 20.16
Males 79.84
Percentage share of the total population
Females 0.493
Males 0.507

Ratio of female non-agricultural wages to male non-agricultural wages: 75.97%
Adjusted real GDP per capita: PPP$3,390

Ratio of the female wage to the average wage (W) and the male wage to the average wage (W)
$W = 0.2016(0.7597) + 0.7984(1) = 0.9516$
Female wage to average wage: $0.7597/0.9516 = 0.7983$
Male wage to average wage: $1.0000/0.9516 = 1.0509$

Share of earned income
Note: [(female wage/average wage) x female share of economically active population] + [(male wage/average wage) x male share of economically active population] = 1.

Females $0.7983 \times 0.2016 = 0.1609$
Males $1.0509 \times 0.7984 = 0.8391$

Female and male proportional income shares
Females $0.1609/0.493 = 0.3264$
Males $0.8391/0.507 = 1.6550$

STEP THREE
Applying the GESI formula
Note: We assume that ϵ, the parameter of inequality aversion, equals 2.

The equally distributed income index
$[0.493(0.3264)^{1-\epsilon} + 0.507(1.6550)^{1-\epsilon}]^{1/(1-\epsilon)} = 0.550$
$0.550 \times 3,390 = 1,865$
$(1,865 - 100)/(5,448 - 100) = 0.330$

The equally distributed index of educational attainment
$[0.497(0.790)^{1-\epsilon} + 0.503(0.817)^{1-\epsilon}]^{1/(1-\epsilon)} = 0.804$

The equally distributed index of life expectancy
$[0.497(0.740)^{1-\epsilon} + 0.503(0.760)^{1-\epsilon}]^{1/(1-\epsilon)} = 0.750$

STEP FOUR
Computing the gender-related development index
$1/3(0.330 + 0.804 + 0.750) = 0.628$

The gender empowerment measure

The gender empowerment measure (GEM) uses variables constructed explicitly to measure the relative empowerment of men and women in political and economic spheres of activity.

The first cluster of variables is chosen to reflect economic participation and decision-making power. It includes women's and men's percentage shares of administrative and managerial positions and percentage shares of professional and technical jobs. These are broad, loosely defined occupational categories. Because the relevant population for each is different, we calculate separate indices for each and then add them together.

For each occupational category, we use the population-weighted $(1-\epsilon)$ averaging of the GESI methodology to derive an equally distributed equivalent percentage (EDEP) for both sexes taken together. To be consistent with the methodology for the GDI, we set the value of ϵ—the parameter that registers the degree of aversion to inequality—equal to 2. Given society's aversion to inequality, the EDEP would be as socially valued as the actual unequal percentages of men and women. If there were perfect equality between women and men, the EDEP would equal 50%. The greater the disparity between female and male shares, the lower the EDEP will be relative to 50%. Thus, for indexing purposes, 50% is our maximum value and 0% our minimum value. After indexing, we add the two categories of occupations together, giving equal weight to each.

The second variable is chosen to reflect political participation and decision-making power. It is women's and men's percentage shares of parliamentary seats. As before, we do the $(1-\epsilon)$ averaging of these two shares to derive the EDEP, and then index it. The maximum value is 50% and the minimum value is 0%, just as for economic participation and decision-making power. (In fact, any zeroes are set equal to a small fraction so that the computations can be carried out.)

The variable we choose to reflect power over economic resources is unadjusted real GDP per capita (PPP$). Unlike adjusted real GDP per capita, which is used in both the HDI and the GDI and ranges from $100 to $5,448, unadjusted real GDP per capita ranges from $100 to $40,000. We follow the same procedure as in the GDI of calculating the proportional income shares of women and men to derive an equally distributed proportional income share through (1– ∈) averaging, and then discounting the average unadjusted real GDP per capita of each country by the degree to which this latter ratio is less than 1. If there were equality between women and men, this ratio would be 1 and average unadjusted income would not be discounted. To index discounted unadjusted income, we use $100 as the minimum and $40,000 as the maximum.

As the final step, we simply add the indices for each of our three clusters of variables and divide by 3. This gives us the overall GEM.

Several other indicators could have been chosen to reflect empowerment in political and economic spheres of activity. But many good indicators are not provided by enough countries to allow meaningful international comparisons. More such indicators can be added to the estimate of the GEM in future as countries make them available.

Illustration of the GEM methodology

The steps to construct the GEM are illustrated using Mexico as an example. Statistics for Mexico show that the greatest disparity between women and men is in the political arena and the least is in skilled and economic leadership positions.

In applying the GESI methodology to GEM variables, we set ∈ equal to 2.

STEP ONE

Calculating indices for parliamentary representation and administrative, managerial, professional and technical positions

Percentage share of parliamentary representation
Females 7.27
Males 92.73
Percentage share of administrative and managerial positions
Females 19.37
Males 80.63
Percentage share of professional and technical positions
Females 43.24
Males 56.76
Percentage share of total population
Females 0.501
Males 0.499

Calculating the equally distributed equivalent percentage (EDEP)

Calculating the EDEP for parliamentary representation
$[0.499(92.73)^{1-\epsilon} + 0.501(7.27)^{1-\epsilon}]^{1/(1-\epsilon)} = 13.46$

Calculating the EDEP for administrative and managerial positions
$[0.499(80.63)^{1-\epsilon} + 0.501(19.37)^{1-\epsilon}]^{1/(1-\epsilon)} = 31.20$

Calculating the EDEP for professional and technical positions
$[0.499(56.76)^{1-\epsilon} + 0.501(43.24)^{1-\epsilon}]^{1/(1-\epsilon)} = 49.08$

Indexing the variables

Parliamentary representation
13.46/50 = 0.2692
Administrative and managerial
31.20/50 = 0.6240
Professional and technical
49.08/50 = 0.9816

Computing the combined index for economic participation and decision-making
(0.6240 + 0.9816)/2 = 0.8028

STEP TWO

Calculating the index for share of earned income

Percentage share of the economically active population
Females 27.63
Males 72.37

Ratio of female non-agricultural wages to male non-agricultural wages: 75%
Unadjusted real GDP per capita: PPP$7,300

Ratio of the female wage to the average wage (W) and the male wage to the average wage (W)
W = 0.2763(0.75) + 0.7237(1) = 0.9309
Female wage to average wage: 0.75/0.9309 = 0.8057
Male wage to average wage: 1.00/0.9309 = 1.0742

Share of earned income
Note: [(female wage/average wage) x female share of economically active population] + [(male wage/average wage) x male share of economically active population] = 1.

Females 0.8057 x 0.2763 = 0.2226
Males 1.0742 x 0.7237 = 0.7774

Female and male proportional income shares
Females 0.2226/0.501 = 0.4443
Males 0.7774/0.499 = 1.5579

Calculating the EDEP of the female and male proportional income shares
$[0.499(1.5579)^{1-\epsilon} + 0.501(0.4443)^{1-\epsilon}]^{1/(1-\epsilon)} = 0.6910$

Computing the income index
0.6910 x 7,300 = 5,044
(5,044 – 100)/(40,000 – 100) = 0.1239

STEP THREE

Computing the gender empowerment measure
(0.2692 + 0.8028 + 0.1239)/3 = 0.399

3. Computing the human development index

The HDI is based on three indicators: longevity, as measured by life expectancy at birth; educational attainment, as measured by a combination of adult literacy (two-thirds weight) and combined primary, secondary and tertiary enrolment ratios (one-third weight); and standard of living, as measured by real GDP per capita (PPP$).

For the construction of the index, fixed minimum and maximum values have been established for each of these indicators:

- Life expectancy at birth: 25 years and 85 years
- Adult literacy: 0% and 100%
- Combined enrolment ratio: 0% and 100%
- Real GDP per capita (PPP$): PPP$100 and PPP$40,000.

Since *Human Development Report 1994,* two changes have been made in the construction of the HDI relating to variables and minimum and maximum values. First, the variable of mean years of schooling has been replaced by the combined primary, secondary and tertiary enrolment ratios, mainly because the formula for calculating mean years of schooling is complex and has enormous data requirements. Data on mean years of schooling are not provided by any UN agency or international organization. As a result, estimates must sometimes be used, which are not always acceptable. The combined enrolment ratio overcomes both these problems. It shows the stock of literacy quite easily for those under age 24. And it is based on the work of UNESCO.

Second, the minimum value of income has been revised from PPP$200 to PPP$100. This revision was made because in the construction of the gender-related development index (GDI) for different countries, the minimum observed value of female income of PPP$100 is used as the lower goal post. It is necessary to use this fixed minimum for construction of the overall HDI to maintain consistency between the construction of the HDI and that of the GDI and to ensure comparability between the two indices. For the HDI, the revision is only marginal, and it had little effect on HDI values.

For any component of the HDI, individual indices can be computed according to the general formula:

$$\text{Index} = \frac{\text{Actual } x_i \text{ value} - \text{minimum } x_i \text{ value}}{\text{Maximum } x_i \text{ value} - \text{minimum } x_i \text{ value}}$$

If, for example, the life expectancy at birth in a country is 65 years, the index of life expectancy for this country would be:

$$\text{Life expectancy index} = \frac{65-25}{85-25} = \frac{40}{60} = 0.667$$

The construction of the income index is a little more complex. As explained in chapter 1, the world average income of PPP$5,120 in 1992 is taken as the threshold level (y^*) and any income above this level is discounted using the following formula for the utility of income:

$$\begin{aligned} W(y) &= y^* \text{ for } 0 < y < y^* \\ &= y^* + 2[(y - y^*)^{1/2}] \text{ for } y^* \le y \le 2y^* \\ &= y^* + 2(y^{*1/2}) + 3[(y - 2y^*)^{1/3}] \text{ for } 2y^* \le y \le 3y^*. \end{aligned}$$

To calculate the discounted value of the maximum income of PPP$40,000, the following formula is used:

$$\begin{aligned} W(y) = \; & y^* + 2(y^{*1/2}) + 3(y^{*1/3}) + 4(y^{*1/4}) + 5(y^{*1/5}) \\ & + 6(y^{*1/6}) + 7(y^{*1/7}) + 8[(40{,}000 - 7y^*)^{1/8}] \end{aligned}$$

This is because PPP$40,000 is between $7y^*$ and $8y^*$. With the above formulation, the discounted value of the maximum income of PPP$40,000 is PPP$5,448.

The actual construction of the HDI is illustrated with two examples—Greece, an industrial country, and Gabon, a developing country.

Country	Life expectancy (years)	Adult literacy (%)	Combined enrolment ratio (%)	Real GDP per capita (PPP$)
Greece	77.6	93.8	78	8,310
Gabon	53.5	58.9	47	3,913

Life expectancy index

$$\text{Greece} = \frac{77.6-25}{85-25} = \frac{52.6}{60} = 0.876$$

$$\text{Gabon} = \frac{53.5-25}{85-25} = \frac{28.5}{60} = 0.475$$

Adult literacy index

$$\text{Greece} = \frac{93.8-0}{100-0} = \frac{93.8}{100} = 0.938$$

$$\text{Gabon} = \frac{58.9-0}{100-0} = \frac{58.9}{100} = 0.589$$

Combined primary, secondary and tertiary enrolment ratio index

$$\text{Greece} = \frac{78-0}{100-0} = 0.780$$

$$\text{Gabon} = \frac{47-0}{100-0} = 0.470$$

Educational attainment index

$$\text{Greece} = [2(0.938) + 1(0.780)] \div 3 = 0.885$$

$$\text{Gabon} = [2(0.589) + 1(0.470)] \div 3 = 0.549$$

Adjusted real GDP per capita (PPP$) index

Greece's real GDP per capita, at PPP$8,310, is above—but less than twice—the threshold. Thus, the adjusted

The method of income discounting used in technical note 3 is the same as that used in *Human Development Report 1992.* It draws on the work of Meghnad Desai.

real GDP per capita for Greece would be PPP$5,233 because 5,233 = [5,120 + 2(8,310 – 5,120)1/2].

Gabon's real GDP per capita income, at PPP$3,913, is less than the threshold, so it needs no adjustment.

The adjusted real GDP per capita (PPP$) index for Greece and Gabon would be:

$$\text{Greece} = \frac{5{,}233 - 100}{5{,}448 - 100} = \frac{5{,}133}{5{,}348} = 0.960$$

$$\text{Gabon} = \frac{3{,}913 - 100}{5{,}449 - 100} = \frac{3{,}813}{5{,}348} = 0.713$$

Human development index

The HDI is a simple average of the life expectancy index, educational attainment index and the adjusted real GDP per capita (PPP$) index. It is calculated by dividing the sum of these three indices by 3. The HDI of both Greece and Gabon are calculated using this formula:

Country	Life expectancy index	Educational attainment index	Adjusted real GDP per capita (PPP$) index	Σ	HDI
Greece	0.876	0.885	0.960	2.721	0.907
Gabon	0.475	0.549	0.713	1.737	0.579