Technical note 1. Properties of the human poverty index

This technical note states, establishes and discusses some important properties of the human poverty index. Intended as an aid to understanding the index, these properties are derived with respect to a more general definition of the human poverty index $P(\alpha)$ than that actually used in this Report. This allows the possibility that the weights on the three poverty subindices may differ, so that $P(\alpha)$ is a weighted mean of order α of P_1, P_2 and P_3.

Thus, letting $w_i > 0$ be the weight on $P_i (\geq 0)$, for $i = 1, 2, 3$, we define the generalized mean $P(\alpha)$ as

$$(1) \qquad P(\alpha) = \left(\frac{w_1 P_1^\alpha + w_2 P_2^\alpha + w_3 P_3^\alpha}{w_1 + w_2 + w_3} \right)^{1/\alpha}.$$

The weighted mean reduces to the ordinary mean of order α when $w_i = 1$ for every i. With $w_1 = w_2 = w_3 = 1$, we have simply

$$(2) \qquad P(\alpha) = \left[\left(\frac{1}{3}\right) P_1^\alpha + \left(\frac{1}{3}\right) P_2^\alpha + \left(\frac{1}{3}\right) P_3^\alpha \right]^{1/\alpha}.$$

The mean of order 1 ($\alpha = 1$) is the simple weighted or unweighted arithmetic mean of P_1, P_2 and P_3. Thus

$$\begin{aligned} P(1) &= \frac{w_1 P_1 + w_2 P_2 + w_3 P_3}{w_1 + w_2 + w_3} \\ &= \frac{1}{3}\left(P_1 + P_2 + P_3\right) \qquad \text{when } w_i = 1 \text{ for every } i. \end{aligned}$$

Can the human poverty index $P(\alpha)$ be interpreted as a headcount or incidence of poverty? While P_1, P_2 and P_3 are the headcount or incidence of poverty in each of three separate dimensions, $P(\alpha)$ cannot be generally thought of as the headcount ratio with respect to a poverty line (hyperplane) drawn in the product space of the three variables. Instead, $P(\alpha)$ is an average, albeit of order α, of the three subindices P_1, P_2 and P_3. If the incidence of poverty happened to be the *same* in every dimension, then $P(\alpha)$ would clearly be equal to this common number, since

$$\left[\frac{w_1 P(\alpha)^\alpha + w_2 P(\alpha)^\alpha + w_3 P(\alpha)^\alpha}{w_1 + w_2 + w_3} \right]^{1/\alpha} = P(\alpha) = \left(\frac{w_1 P_1^\alpha + w_2 P_2^\alpha + w_3 P_3^\alpha}{w_1 + w_2 + w_3} \right)^{1/\alpha}.$$

This observation allows us to interpret $P(\alpha)$ as the degree of overall poverty that is equivalent to having a headcount ratio of $P(\alpha)$% in every dimension.

The first property of $P(\alpha)$ that we establish is central to understanding it as a mean of P_1, P_2 and P_3. This property is that $P(\alpha)$ always lies between the smallest and largest values of P_i, for $i = 1, 2, 3$.

PROPOSITION 1.

$$\min\left\{P_1, P_2, P_3\right\} \leq P(\alpha) \leq \max\left\{P_1, P_2, P_3\right\}.$$

PROOF. By definition of $P(\alpha)$, we have

$$(3) \quad P(\alpha)^\alpha = \frac{w_1}{w_1 + w_2 + w_3} P_1^\alpha + \frac{w_2}{w_1 + w_2 + w_3} P_2^\alpha + \frac{w_3}{w_1 + w_2 + w_3} P_3^\alpha.$$

But for each $i = 1, 2, 3$,

$$\min\left\{P_1, P_2, P_3\right\} \leq P_i \leq \max\left\{P_1, P_2, P_3\right\}.$$

Therefore, since $\alpha > 0$,

$$\left[\min\left\{P_1, P_2, P_3\right\}\right]^\alpha \leq P_i^\alpha \leq \left[\max\left\{P_1, P_2, P_3\right\}\right]^\alpha.$$

Using the right-hand-side inequality for each P_i^α in equation 3 gives

$$\begin{aligned} P(\alpha)^\alpha &\leq \frac{w_1 + w_2 + w_3}{w_1 + w_2 + w_3}\left[\max\left\{P_1, P_2, P_3\right\}\right]^\alpha \\ &= \left[\max\left\{P_1, P_2, P_3\right\}\right]^\alpha. \end{aligned}$$

Similarly,

$$P(\alpha)^\alpha \geq \left[\min\left\{P_1, P_2, P_3\right\}\right]^\alpha.$$

Hence

$$\left[\min\left\{P_1, P_2, P_3\right\}\right]^\alpha \leq P(\alpha)^\alpha \leq \left[\max\left\{P_1, P_2, P_3\right\}\right]^\alpha.$$

Since $\alpha > 0$, it follows that

$$\min\left\{P_1, P_2, P_3\right\} \leq P(\alpha) \leq \max\left\{P_1, P_2, P_3\right\}. \quad \square$$

The generalized mean $P(\alpha)$ is constructed for values of $\alpha \geq 1$. As shown, its limiting value when $\alpha = 1$ is simply the arithmetic mean of P_1, P_2 and P_3. In proposition 7 we show that the larger α is, the larger $P(\alpha)$ will be. For expositional reasons, it is convenient to demonstrate at this stage that as α tends to infinity, the limiting value of $P(\alpha)$ is max $\{P_1, P_2, P_3\}$.

PROPOSITION 2. As $\alpha \to \infty$,

$$P(\alpha) \to \max\left\{P_1, P_2, P_3\right\}.$$

PROOF. Let P_k be the largest—or in the case of ties, one of the largest—P_i, for $i = 1, 2, 3$. Thus

$$P_k = \max\left\{P_1, P_2, P_3\right\}.$$

Then from proposition 1, for any $\alpha > 0$, we have

$$(4) \qquad P(\alpha) \leq P_k = \max\left\{P_1, P_2, P_3\right\}.$$

Now

$$\begin{aligned} P(\alpha)^\alpha &= \frac{w_1}{w_1 + w_2 + w_3} P_1^\alpha + \frac{w_2}{w_1 + w_2 + w_3} P_2^\alpha + \frac{w_3}{w_1 + w_2 + w_3} P_3^\alpha \\ &\geq \frac{w_k}{w_1 + w_2 + w_3} P_k^\alpha \qquad \text{since } P_k \text{ is one of } P_1, P_2 \text{ or } P_3. \end{aligned}$$

Technical note 1 is from the *Human Development Report 1997* background paper by Sudhir Anand and Amartya K. Sen, "Concepts of Human Development and Poverty: A Multidimensional Perspective."

Therefore, since $\alpha > 0$,

$$P(\alpha) \geq \left(\frac{w_k}{w_1 + w_2 + w_3} \right)^{1/\alpha} P_k.$$

Letting $\alpha \to \infty$, $\left(\frac{w_k}{w_1 + w_2 + w_3} \right)^{1/\alpha} \to 1$,

so that $\lim_{\alpha \to \infty} P(\alpha) \geq P_k$.

But from equation 4 we also have

$$\lim_{\alpha \to \infty} P(\alpha) \leq P_k.$$

Hence

$$\lim_{\alpha \to \infty} P(\alpha) = P_k = \max\left\{P_1, P_2, P_3\right\}. \quad \square$$

The next property of $P(\alpha)$ that we demonstrate is that the index is homogeneous of degree 1 in the subindices P_1, P_2 and P_3. In other words, if the incidence of poverty in each dimension is halved (multiplied by $\lambda > 0$), the value of the aggregate index $P(\alpha)$ will be halved (changed to λ multiplied by $P(\alpha)$).

PROPOSITION 3. $P(\alpha)$ is homogeneous of degree 1 in (P_1, P_2, P_3).

PROOF. Let $\lambda > 0$ be a scalar number, and let $P(\alpha)$ be the value of the human poverty index corresponding to (P_1, P_2, P_3).

Then

$$P(\alpha) = \left(\frac{w_1 P_1^\alpha + w_2 P_2^\alpha + w_3 P_3^\alpha}{w_1 + w_2 + w_3} \right)^{1/\alpha}.$$

The value of the human poverty index corresponding to $(\lambda P_1, \lambda P_2, \lambda P_3)$ is then given by

$$\left[\frac{w_1(\lambda P_1)^\alpha + w_2(\lambda P_2)^\alpha + w_3(\lambda P_3)^\alpha}{w_1 + w_2 + w_3} \right]^{1/\alpha} = \left[\frac{\lambda^\alpha(w_1 P_1^\alpha + w_2 P_2^\alpha + w_3 P_3^\alpha)}{w_1 + w_2 + w_3} \right]^{1/\alpha}$$
$$= \lambda P(\alpha). \quad \square$$

The next property of $P(\alpha)$ that we derive is that $P(\alpha)$ is monotonic increasing in each P_i, for $i = 1, 2, 3$.

PROPOSITION 4. For each $i = 1, 2, 3$,

$$\frac{\partial P(\alpha)}{\partial P_i} > 0.$$

PROOF. From the definition of the generalized mean $P(\alpha)$ we have

$$(w_1 + w_2 + w_3)\, P(\alpha)^\alpha = w_1 P_1^\alpha + w_2 P_2^\alpha + w_3 P_3^\alpha.$$

Differentiating partially with respect to P_i,

$$(w_1 + w_2 + w_3)\, \alpha P(\alpha)^{\alpha-1} \frac{\partial P(\alpha)}{\partial P_i} = w_i \alpha P_i^{\alpha-1}.$$

Therefore

$$(5) \qquad \frac{\partial P(\alpha)}{\partial P_i} = \frac{w_i}{w_1 + w_2 + w_3} \left[\frac{P_i}{P(\alpha)} \right]^{\alpha-1}$$
$$> 0 \quad \text{because } w_i > 0. \quad \square$$

In the unit weights case ($w_i = 1$, for $i = 1, 2, 3$) this reduces to

$$\frac{\partial P(\alpha)}{\partial P_i} = \frac{1}{3} \left[\frac{P_i}{P(\alpha)} \right]^{\alpha-1}.$$

Moreover, for $\alpha = 1$, so that $P(1)$ is simply the weighted or unweighted arithmetic mean of P_i, we have

$$\frac{\partial P(1)}{\partial P_i} = \frac{w_i}{w_1 + w_2 + w_3}$$

or

$$\frac{\partial P(1)}{\partial P_i} = \frac{1}{3}.$$

For an aggregate poverty index $P(\alpha)$ composed of distinct poverty subindices P_1, P_2 and P_3, it seems clearly desirable that $P(\alpha)$ should be increasing in each P_i. Also desirable is that $P(\alpha)$ should increase at an increasing rate in P_i—in other words, that $P(\alpha)$ should be convex with respect to P_i. This is equivalent to saying that $P(\alpha)$ decreases with reductions in P_i, and at a diminishing rate. The next proposition establishes that our aggregator function $P(\alpha)$, for $\alpha > 1$, does satisfy this property.

PROPOSITION 5. For each $i = 1, 2, 3$,

$$\frac{\partial^2 P(\alpha)}{\partial P_i^2} > 0.$$

PROOF.

$$\frac{\partial^2 P(\alpha)}{\partial P_i^2} = \frac{\partial}{\partial P_i} \left[\frac{\partial P(\alpha)}{\partial P_i} \right]$$
$$= \frac{w_i}{w_1 + w_2 + w_3} \frac{\partial}{\partial P_i} \left\{ \left[\frac{P_i}{P(\alpha)} \right]^{\alpha-1} \right\}$$

from equation 5.

Now

$$\frac{\partial}{\partial P_i} \left[\frac{P_i}{P(\alpha)} \right]^{\alpha-1} = (\alpha - 1) \left[\frac{P_i}{P(\alpha)} \right]^{\alpha-2} \frac{\partial}{\partial P_i} \left[\frac{P_i}{P(\alpha)} \right]$$
$$= (\alpha - 1) \left[\frac{P_i}{P(\alpha)} \right]^{\alpha-2} \left[P(\alpha) - P_i \frac{\partial P(\alpha)}{\partial P_i} \right] \Big/ P(\alpha)^2$$
$$= (\alpha - 1) \frac{P_i^{\alpha-2}}{P(\alpha)^\alpha} \left[P(\alpha) - \frac{P_i w_i P_i^{\alpha-1}}{(w_1 + w_2 + w_3)\, P(\alpha)^{\alpha-1}} \right]$$

substituting for $\frac{\partial P(\alpha)}{\partial P_i}$ from equation 5

$$= \frac{(\alpha - 1) P_i^{\alpha-2}}{P(\alpha)^\alpha} \left[\frac{(w_1 + w_2 + w_3) P(\alpha)^\alpha - w_i P_i^\alpha}{(w_1 + w_2 + w_3) P(\alpha)^{\alpha-1}} \right].$$

Hence

$$\frac{\partial^2 P(\alpha)}{\partial P_i^2} = \frac{w_i P_i^{\alpha-2}(\alpha-1)}{(w_1+w_2+w_3)^2 P(\alpha)^{2\alpha-1}}\left[(w_1+w_2+w_3)P(\alpha)^{\alpha} - w_i P_i^{\alpha}\right]$$
$$>0$$

because $\alpha > 1$ and

$$(w_1+w_2+w_3)P(\alpha)^{\alpha} - w_i P_i^{\alpha} = \sum_{j\neq i} w_j P_j^{\alpha} > 0. \quad \square$$

The next property we consider is the effect on the aggregate index $P(\alpha)$ of increasing the weight w_i on a particular poverty subindex P_i. We expect that increasing the weight on the largest subindex, max $\{P_1, P_2, P_3\}$, will increase $P(\alpha)$, while increasing the weight on the smallest subindex, min $\{P_1, P_2, P_3\}$, will reduce $P(\alpha)$. But what would be the effect of increasing the weight on a middle P_i? The answer depends on the relationship between P_i and $P(\alpha)$.

Proposition 6. For any i,

$$\frac{\partial P(\alpha)}{\partial w_i} \gtreqqless 0 \text{ as } P_i \gtreqqless P(\alpha).$$

Proof. From the definition of $P(\alpha)$ we have

$$(w_1+w_2+w_3)P(\alpha)^{\alpha} = w_1 P_1^{\alpha} + w_2 P_2^{\alpha} + w_3 P_3^{\alpha}.$$

Differentiating both sides partially with respect to w_i,

$$(w_1+w_2+w_3)\alpha P(\alpha)^{\alpha-1}\frac{\partial P(\alpha)}{\partial w_i} + P(\alpha)^{\alpha} = P_i^{\alpha}.$$

Therefore

$$(w_1+w_2+w_3)\alpha P(\alpha)^{\alpha-1}\frac{\partial P(\alpha)}{\partial w_i} = P_i^{\alpha} - P(\alpha)^{\alpha}.$$

Hence, since $\alpha > 0$,

$$\frac{\partial P(\alpha)}{\partial w_i} \gtreqqless 0 \text{ as } P_i^{\alpha} \gtreqqless P(\alpha)^{\alpha},$$

that is,

$$\text{as } P_i \gtreqqless P(\alpha). \quad \square$$

For $\alpha = 1$ we have

$$\frac{\partial P(1)}{\partial w_i} = \frac{1}{w_1+w_2+w_3}\left[P_i - P(1)\right]$$
$$\gtreqqless 0 \text{ as } P_i \gtreqqless P(1).$$

The next property we consider is the effect on $P(\alpha)$ of raising the parameter value α for given values of the subindices P_i, for $i = 1, 2, 3$. It shows that the value of the aggregate index will be higher when a higher-order mean is formed of P_1, P_2 and P_3. In particular, a mean of order $\alpha > 1$ will result in a $P(\alpha)$ that is greater than $P(1)$, the simple arithmetic mean of P_1, P_2 and P_3.

Proposition 7. For given P_1, P_2 and P_3 that are not equal, if $\alpha > \gamma > 0$, then $P(\alpha) > P(\gamma)$.

Proof. Let $\alpha > \gamma > 0$. By definition of $P(\alpha)$ and $P(\gamma)$, we have

$$P(\alpha)^{\alpha} = \frac{w_1}{w_1+w_2+w_3}P_1^{\alpha} + \frac{w_2}{w_1+w_2+w_3}P_2^{\alpha} + \frac{w_3}{w_1+w_2+w_3}P_3^{\alpha}$$

and

$$P(\gamma)^{\gamma} = \frac{w_1}{w_1+w_2+w_3}P_1^{\gamma} + \frac{w_2}{w_1+w_2+w_3}P_2^{\gamma} + \frac{w_3}{w_1+w_2+w_3}P_3^{\gamma}.$$

Raising both sides of the second equation to the power (α/γ) (> 1 because $\alpha > \gamma > 0$),

$$\left[P(\gamma)^{\gamma}\right]^{\alpha/\gamma} = \left(\frac{w_1}{w_1+w_2+w_3}P_1^{\gamma} + \frac{w_2}{w_1+w_2+w_3}P_2^{\gamma} + \frac{w_3}{w_1+w_2+w_3}P_3^{\gamma}\right)^{\alpha/\gamma}.$$

Now $f(x) = x^{\alpha/\gamma}$ is a strictly convex function, since

$$f'(x) = (\alpha/\gamma)\, x^{(\alpha/\gamma)-1}$$

and

$$f''(x) = (\alpha/\gamma)\left[(\alpha/\gamma)-1\right]x^{(\alpha/\gamma)-2}$$
$$>0 \quad \text{because } (\alpha/\gamma) > 1.$$

Hence, by Jensen's inequality applied to strictly convex functions $f(\cdot)$, since P_1, P_2 and P_3 are not equal, we have the strict inequality

$$f\left(\frac{w_1}{w_1+w_2+w_3}P_1^{\gamma} + \frac{w_2}{w_1+w_2+w_3}P_2^{\gamma} + \frac{w_3}{w_1+w_2+w_3}P_3^{\gamma}\right)$$
$$< \frac{w_1}{w_1+w_2+w_3}f(P_1^{\gamma}) + \frac{w_2}{w_1+w_2+w_3}f(P_2^{\gamma}) + \frac{w_3}{w_1+w_2+w_3}f(P_3^{\gamma}).$$

Using the strictly convex function $f(x) = x^{\alpha/\gamma}$ gives

$$\left[P(\gamma)^{\gamma}\right]^{\alpha/\gamma} < \frac{w_1}{w_1+w_2+w_3}P_1^{\alpha} + \frac{w_2}{w_1+w_2+w_3}P_2^{\alpha} + \frac{w_3}{w_1+w_2+w_3}P_3^{\alpha},$$

that is,

$$P(\gamma)^{\alpha} < P(\alpha)^{\alpha}.$$

Since $\alpha > 0$, it follows that

$$P(\gamma) < P(\alpha). \quad \square$$

Letting $\gamma = 1$ and $\alpha > 1$, we have the corollary that

$$P(\alpha) > P(1) = \frac{w_1 P_1 + w_2 P_2 + w_3 P_3}{w_1+w_2+w_3},$$

the simple weighted arithmetic mean of P_1, P_2 and P_3.

We next investigate the "decomposability" of the human poverty index among groups within a country. Suppose the population of a country is divided into m mutually exclusive and exhaustive groups. The groups may be defined in terms of stratum (urban, rural), region (by state, province or district) or gender (male, female). Let n_j be the size of population group j, for $j = 1, 2, \ldots, m$, and let n be the size of the total population of the country. Then

$$n = \sum_{j=1}^{m} n_j.$$

Let P_{1j}, P_{2j} and P_{3j} be the values of the three poverty subindices P_1, P_2 and P_3 for group j, where $j = 1, 2, \ldots, m$. Finally, let $P_j(\alpha)$ denote the mean of order α of P_{1j}, P_{2j} and P_{3j} for group j. By definition, we have

$$P_j(\alpha) = \left(\frac{w_1 P_{1j}^{\alpha} + w_2 P_{2j}^{\alpha} + w_3 P_{3j}^{\alpha}}{w_1 + w_2 + w_3} \right)^{1/\alpha}, \text{ for } j = 1, 2, \ldots, m.$$

What is the relationship between $P(\alpha)$ and the $P_j(\alpha)$ for $j = 1, 2, \ldots, m$? Strict decomposability of the index $P(\alpha)$ would require that $P(\alpha)$ be a population-weighted average of the $P_j(\alpha)$, the population weights being n_j/n. But strict decomposability does not generally obtain.

The relationship between the values of a given subindex for different groups (for example, P_{1j}, for $j = 1, 2, \ldots, m$) and the overall value of the subindex (for example, P_1) is straightforward enough. As the indices are simple headcounts of poverty, we have

$$\sum_{j=1}^{m} \frac{n_j}{n} P_{1j} = P_1,$$

$$\sum_{j=1}^{m} \frac{n_j}{n} P_{2j} = P_2,$$

$$\text{and } \sum_{j=1}^{m} \frac{n_j}{n} P_{3j} = P_3.$$

But when the α-averages of P_{1j}, P_{2j} and P_{3j} are formed for each j to give $P_j(\alpha)$, the population-weighted average of the $P_j(\alpha)$s exceeds $P(\alpha)$.

PROPOSITION 8. For $\alpha \geq 1$,

$$\sum_{j=1}^{m} \frac{n_j}{n} P_j(\alpha) \geq P(\alpha).$$

PROOF. For each $j = 1, 2, \ldots, m$, we have

$$\frac{n_j}{n} P_j(\alpha) = \left[\frac{w_1 \left(\frac{n_j}{n} P_{1j} \right)^{\alpha} + w_2 \left(\frac{n_j}{n} P_{2j} \right)^{\alpha} + w_3 \left(\frac{n_j}{n} P_{3j} \right)^{\alpha}}{w_1 + w_2 + w_3} \right]^{1/\alpha}.$$

Applying Minkowski's inequality (Hardy, Littlewood and Pólya 1952, p. 30) to $(n_j/n)P_{1j}$, $(n_j/n)P_{2j}$, $(n_j/n)P_{3j}$ for $j = 1, 2, \ldots, m$ yields

$$\sum_{j=1}^{m} \left[\frac{w_1 \left(\frac{n_j}{n} P_{1j} \right)^{\alpha} + w_2 \left(\frac{n_j}{n} P_{2j} \right)^{\alpha} + w_3 \left(\frac{n_j}{n} P_{3j} \right)^{\alpha}}{w_1 + w_2 + w_3} \right]^{1/\alpha}$$

$$\geq \left[\frac{w_1 \left(\sum_{j=1}^{m} \frac{n_j}{n} P_{1j} \right)^{\alpha} + w_2 \left(\sum_{j=1}^{m} \frac{n_j}{n} P_{2j} \right)^{\alpha} + w_3 \left(\sum_{j=1}^{m} \frac{n_j}{n} P_{3j} \right)^{\alpha}}{w_1 + w_2 + w_3} \right]^{1/\alpha}.$$

Hence

$$\sum_{j=1}^{m} \frac{n_j}{n} P_j(\alpha) \geq \left(\frac{w_1 P_1^{\alpha} + w_2 P_2^{\alpha} + w_3 P_3^{\alpha}}{w_1 + w_2 + w_3} \right)^{1/\alpha}.$$

Therefore

$$\sum_{j=1}^{m} \frac{n_j}{n} P_j(\alpha) \geq P(\alpha). \quad \square$$

The weak inequality in proposition 8 will be a strict inequality unless either $\alpha = 1$ or (P_{1j}, P_{2j}, P_{3j}) and (P_{1k}, P_{2k}, P_{3k}) are proportional for all j and k.

A simple example with non-proportionality of the group poverty subindices shows why decomposability (equality in proposition 8) does not obtain for $\alpha > 1$. Suppose the population is divided into two mutually exclusive and exhaustive groups $j = 1, 2$ of equal size ($n_1/n = n_2/n = 1/2$), with values of poverty subindices as follows:

$$(P_{11}, P_{21}, P_{31}) = (0.25, 0.5, 0.75),$$
$$\text{and } (P_{12}, P_{22}, P_{32}) = (0.75, 0.5, 0.25).$$

Hence

$$(P_1, P_2, P_3) = (0.5, 0.5, 0.5),$$

and obviously $P(\alpha) = 0.5$.

Now for group 1

$$P_1(\alpha) = [(1/3)(0.25)^{\alpha} + (1/3)(0.5)^{\alpha} + (1/3)(0.75)^{\alpha}]^{1/\alpha}$$
$$> 0.5, \quad \text{by proposition 7 since } \alpha > 1,$$

and for group 2

$$P_2(\alpha) = [(1/3)(0.75)^{\alpha} + (1/3)(0.5)^{\alpha} + (1/3)(0.25)^{\alpha}]^{1/\alpha}$$
$$> 0.5, \quad \text{by proposition 7 since } \alpha > 1.$$

Therefore

$$(1/2)P_1(\alpha) + (1/2)P_2(\alpha) > (1/2)(0.5) + (1/2)(0.5)$$
$$= 0.5$$
$$= P(\alpha).$$

Taking the group arithmetic means of each poverty subindex tends to reduce or leave unchanged the relative disparity among the three poverty subindices. As a result of this feature the α-average of the arithmetic means of group subindices is smaller than the arithmetic mean of α-averages of group subindices.

Finally, for a given value of $\alpha (\geq 1)$, we discuss the degree of substitutability between the poverty subindices P_1, P_2 and P_3 in the aggregate measure $P(\alpha)$. The elasticity of substitution between, say, P_1 and P_2 along an iso-$P(\alpha)$ curve (holding P_3 constant) is defined as the percentage change in (P_1/P_2) for a unit percentage change in the slope of the tangent along this curve (projected onto P_1-P_2 space at the given value of P_3). For the index $P(\alpha)$ the elasticity of substitution is constant along each level set of $P(\alpha)$ and the same for different level sets. By proposition 3, $P(\alpha)$ is homogeneous of degree 1 in (P_1, P_2, P_3), and therefore its level sets are homothetic.

PROPOSITION 9. The elasticity of substitution σ between any two subindices of $P(\alpha)$, that is, between any two of P_1, P_2 and P_3, is constant and equal to $1/(\alpha-1)$.

PROOF. Consider the elasticity of substitution between P_1 and P_2, holding P_3 constant. The slope of the tangent along an iso-$P(\alpha)$ curve in P_1-P_2 space is given by

$$x = \frac{\partial P(\alpha)}{\partial P_1} \bigg/ \frac{\partial P(\alpha)}{\partial P_2}.$$

By definition, the elasticity of substitution σ between P_1 and P_2 is

$$\frac{\partial \log(P_1/P_2)}{\partial \log x}.$$

From equation 5 in proposition 4 we have

$$\frac{\partial P(\alpha)}{\partial P_1} \bigg/ \frac{\partial P(\alpha)}{\partial P_2} = \frac{w_1}{w_2}\left(\frac{P_1}{P_2}\right)^{\alpha-1} = x.$$

Therefore

$$\frac{P_1}{P_2} = \left(\frac{w_2}{w_1}\right)^{1/(\alpha-1)} x^{1/(\alpha-1)}$$

and

$$\log\left(\frac{P_1}{P_2}\right) = \frac{1}{\alpha-1}\log\left(\frac{w_2}{w_1}\right) + \frac{1}{\alpha-1}\log x.$$

Hence the elasticity of substitution

$$\sigma = \frac{\partial \log(P_1/P_2)}{\partial \log x} = \frac{1}{\alpha-1}. \quad \square$$

Thus, if $\alpha = 1$, there is infinite, or perfect, substitutability between P_1 and P_2. And as $\alpha \to \infty$, there is no substitutability between P_1 and P_2. As α increases from 1, the elasticity of substitution decreases monotonically from ∞ to 0.

If we choose $\alpha = 1$ (the case of perfect substitutability), the aggregate index $P(\alpha)$ is the simple arithmetic mean of the three subindices P_1, P_2, P_3. As α tends to infinity, the substitutability becomes zero, and the aggregate index tends to the maximum of the three subindices, max $\{P_1, P_2, P_3\}$. In general, the elasticity of substitution between any two of the subindices, holding the other constant, is $\sigma = 1/(\alpha-1)$.

With $\alpha = 1$ and infinite substitutability, the impact on $P(\alpha)$ from a unit increase (or decrease) of any subindex is the same, irrespective of the level of deprivation in the different dimensions. This contradicts the usual assumption that as the extent of deprivation in any dimension increases (given the others), the weight on further additions to deprivation in that dimension should also increase. For this we need $\alpha > 1$. The value of α also influences, correspondingly, the relative weight to be placed on deprivation in the different dimensions. Consider, for example, $P_1 = 60\%$ and $P_2 = 30\%$ (with, say, $P_3 = 45\%$). In this case, for any α the relative impact of a unit increase in P_1 compared with a unit increase in P_2, which is given in general by $(P_1/P_2)^{\alpha-1}$, equals $2^{\alpha-1}$. With $\alpha = 1$, the relative impact is given by 1. As was remarked earlier, as α tends to infinity, P_1 becomes the only determinant of $P(\alpha)$, so that its impact is infinitely larger than that of a unit increase in P_2, which has, in this case, no impact at all.

The relative impact increases as α is raised from 1. With $\alpha = 3$, the relative impact is 4, giving the dimension of doubly greater deprivation (P_1) much greater weight. The relative impact rises very fast with the raising of α, as is clear from the formula. For $\alpha = 5$, the relative impact of a unit increase in P_1 is as much as 16 times that of a unit increase in P_2.

For calculating the human poverty index, $\alpha = 3$ has been chosen. This gives an elasticity of substitution of $^1/_2$ and places greater weight on those dimensions in which deprivation is larger. It does not, however, have the extremism of zero substitutability (given by α tending to infinity), nor the very high values of relative impact that are generated as α is raised (increasing the relative impact, in the case discussed above, from 4 to 16 as α goes from 3 to 5). There is an inescapable arbitrariness in the choice of α. The right way to deal with this issue is to explain clearly what is being assumed, as has been attempted here, so that public criticism of this assumption is possible.

As a matter of intellectual continuity, it should be mentioned that the value of $\alpha = 3$ corresponds exactly to the weighting used to calculate the gender-related development index (GDI).

Technical note 2. Computing the indices

The human development index

The HDI is based on three indicators: longevity, as measured by life expectancy at birth; educational attainment, as measured by a combination of adult literacy (two-thirds weight) and combined primary, secondary and tertiary enrolment ratios (one-third weight); and standard of living, as measured by real GDP per capita (PPP$).

For the construction of the index, fixed minimum and maximum values have been established for each of these indicators:

- Life expectancy at birth: 25 years and 85 years
- Adult literacy: 0% and 100%
- Combined gross enrolment ratio: 0% and 100%
- Real GDP per capita (PPP$): $100 and $40,000 (PPP$).

For any component of the HDI, individual indices can be computed according to the general formula:

$$\text{Index} = \frac{\text{Actual } x_i \text{ value} - \text{minimum } x_i \text{ value}}{\text{Maximum } x_i \text{ value} - \text{minimum } x_i \text{ value}} .$$

If, for example, the life expectancy at birth in a country is 65 years, then the index of life expectancy for this country would be

$$\frac{65-25}{85-25} = \frac{40}{60} = 0.667.$$

The construction of the income index is a little more complex. The world average income of $5,835 (PPP$) in 1994 is taken as the threshold level (y^*), and any income above this level is discounted using the following formulation based on Atkinson's formula for the utility of income:

$$\begin{aligned} W(y) &= y^* \text{ for } 0 < y < y^* \\ &= y^* + 2[(y - y^*)^{1/2}] \text{ for } y^* \le y \le 2y^* \\ &= y^* + 2(y^{*1/2}) + 3[(y - 2y^*)^{1/3}] \text{ for } 2y^* \le y \le 3y^* \\ y &= y^* + 2(y^{*1/2}) + 3[(y - 2y^*)^{1/3}] + n\{[1 - (n-1)y^*]\}^{1/n} \\ &\qquad \text{for } (n-1)y^* \le y \le ny^*. \end{aligned}$$

To calculate the discounted value of the maximum income of $40,000 (PPP$), the following form of Atkinson's formula is used:

$$\begin{aligned} W(y) = {} & y^* + 2(y^{*1/2}) + 3(y^{*1/3}) + 4(y^{*1/4}) + 5(y^{*1/5}) \\ & + 6(y^{*1/6}) + 7(y^{*1/7}) + 8[(40{,}000 - 7y^*)^{1/8}] . \end{aligned}$$

This is because $40,000 (PPP$) is between $7y^*$ and $8y^*$. With the above formulation, the discounted value of the maximum income of $40,000 (PPP$) is $6,154 (PPP$).

The construction of the HDI is illustrated with two examples—Greece and Gabon, an industrial and a developing country.

Country	Life expectancy (years)	Adult literacy rate (%)	Combined enrolment ratio (%)	Real GDP per capita (PPP$)
Greece	77.8	96.7	82	11,265
Gabon	54.1	62.6	60	3,641

Life expectancy index

$$\text{Greece} = \frac{77.8-25}{85-25} = \frac{52.8}{60} = 0.880$$

$$\text{Gabon} = \frac{54.1-25}{85-25} = \frac{29.1}{60} = 0.485$$

Adult literacy index

$$\text{Greece} = \frac{96.7-0}{100-0} = \frac{96.7}{100} = 0.967$$

$$\text{Gabon} = \frac{62.6-0}{100-0} = \frac{62.6}{100} = 0.626$$

Combined primary, secondary and tertiary enrolment ratio index

$$\text{Greece} = \frac{82-0}{100-0} = 0.820$$

$$\text{Gabon} = \frac{60-0}{100-0} = 0.600$$

Educational attainment index

Greece = [2(0.967) + 1(0.820)] ÷ 3 = 0.918

Gabon = [2(0.625) + 1(0.600)] ÷ 3 = 0.617

Adjusted real GDP per capita (PPP$) index

Greece's real GDP per capita at $11,265 (PPP$) is above the threshold level, but less than twice the threshold. Thus the adjusted real GDP per capita for Greece would be $5,982 (PPP$) because $5,982 = [5,835 + 2(11,265 – 5,835)^{1/2}]$.

Gabon's real GDP per capita at $3,641 (PPP$) is less than the threshold level, so it needs no adjustment.

Thus the adjusted real GDP per capita (PPP$) indices for Greece and Gabon would be:

$$\text{Greece} = \frac{5{,}982-100}{6{,}154-100} = \frac{5{,}882}{6{,}054} = 0.972$$

$$\text{Gabon} = \frac{3{,}641-100}{6{,}154-100} = \frac{3{,}541}{6{,}054} = 0.584$$

Human development index

The HDI is a simple average of the life expectancy index, educational attainment index and adjusted real GDP per capita (PPP$) index, and so is derived by dividing the sum of these three indices by 3.

Country	Life expectancy index	Educational attainment index	Adjusted real GDP per capita (PPP$) index	HDI
Greece	0.880	0.918	0.972	0.923
Gabon	0.485	0.617	0.584	0.562

The gender-related development index and the gender empowerment measure

For comparisons among countries, the GDI and the GEM are limited to data widely available in international data sets. For this year's Report we have endeavoured to use the most recent, reliable and internally consistent data. Collecting more extensive and more reliable gender-disaggregated data is a challenge that the international community should squarely face. We continue to publish results on the GDI and the GEM—based on the best available estimates—in the expectation that it will help increase the demand for such data.

The gender-related development index

The GDI uses the same variables as the HDI. The difference is that the GDI adjusts the average achievement of each country in life expectancy, educational attainment and income in accordance with the disparity in achievement between women and men. (For a detailed explanation of the GDI methodology see technical note 1 in *Human Development Report 1995*). For this gender-sensitive adjustment we use a weighting formula that expresses a moderate aversion to inequality, setting the weighting parameter, $\in$, equal to 2. This is the harmonic mean of the male and female values.

The GDI also adjusts the maximum and minimum values for life expectancy, to account for the fact that women tend to live longer than men. For women the maximum value is 87.5 years and the minimum value 27.5 years; for men the corresponding values are 82.5 and 22.5 years.

Calculating the index for income is fairly complex. Female and male shares of earned income are derived from data on the ratio of the average female wage to the average male wage and the female and male percentage shares of the economically active population aged 15 and above. Where data on the wage ratio are not available, we use a value of 75%, the weighted mean of the wage ratio for all countries with wage data. Before income is indexed, the average adjusted real GDP per capita of each country is discounted on the basis of the disparity in the female and male shares of earned income in proportion to the female and male population shares.

The indices for life expectancy, educational attainment and income are added together with equal weight to derive the final GDI value.

Illustration of the GDI methodology

We choose Norway to illustrate the steps for calculating the gender-related development index. The parameter of inequality aversion, $\in$, equals 2. (Any discrepancies in results are due to rounding.)

Percentage share of total population
Females 51
Males 49
Life expectancy at birth (years)
Females 80.4
Males 74.6
Adult literacy rate (percent)
Females 99
Males 99
Combined primary, secondary and tertiary gross enrolment ratio (percent)
Females 93
Males 92

STEP ONE
Computing the equally distributed life expectancy index

Life expectancy index
Females $(80.4 - 27.5)/60 = 0.882$
Males $(74.6 - 22.5)/60 = 0.868$

The equally distributed life expectancy index
{[(female population share × (female life expectancy index)$^{-1}$] + [male population share × (male life expectancy index)$^{-1}$]}$^{-1}$
$[0.51(0.882)^{-1} + 0.49(0.868)^{-1}]^{-1} = 0.875$

STEP TWO
Computing the equally distributed educational attainment index

Adult literacy index
Females $(99 - 0)/100 = 0.990$
Males $(99 - 0)/100 = 0.990$
Combined gross enrolment index
Females $(93 - 0)/100 = 0.930$
Males $(92 - 0)/100 = 0.920$
Educational attainment index
2/3(adult literacy index) + 1/3(combined gross enrolment index)
Females $2/3(0.990) + 1/3(0.930) = 0.970$
Males $2/3(0.990) + 1/3(0.920) = 0.967$

The equally distributed educational attainment index
{[female population share × (educational attainment index)$^{-1}$] + [male population share × (educational attainment index)$^{-1}$]}$^{-1}$
$[0.51(0.970)^{-1} + 0.49(0.967)^{-1}]^{-1} = 0.968$

STEP THREE
Computing the equally distributed income index

Percentage share of the economically active population
Females 45.5
Males 54.5

Ratio of female non-agricultural wage to male non-agricultural wage: 0.870

Adjusted real GDP per capita: PPP$6,073 (see the section above on the HDI)

A. Computing proportional income shares

Average wage (W) = (female share of economically active population × female wage) + (male economically active population × 1)
$(0.455 \times 0.870) + (0.545 \times 1) = 0.941$

Female wage to average wage (W)
$0.870/0.941 = 0.925$
Male wage to average wage (W)
$1/0.941 = 1.063$

Share of earned income
Note: [(female wage/average wage) × female share of economically active population] + [(male wage/average wage) × male share of economically active population] = 1
Females
Female wage/female economically active population
$0.9247 \times 0.4553 = 0.4210$
Males
Male wage/male economically active population
$1.063 \times 0.545 = 0.579$

Female and male proportional income shares
Females
Female share of earned income/female population share
$0.421/0.505 = 0.834$
Males
Male share of earned income/male population share
$0.579/0.495 = 1.169$

B. Computing the equally distributed income index

The weighting parameter ($\in = 2$) is applied.

$\{[\text{female population share} \times (\text{female proportional income share})^{-1}] + [\text{male population share} \times (\text{male proportional income share})^{-1}]\}^{-1}$

$[0.505\ (0.834)^{-1} + 0.495\ (1.169)^{-1}]^{-1} = 0.972$

$0.972 \times 6{,}073 = 5{,}903$

$(5{,}903 - 100)/(6{,}154 - 100) = 0.959$

STEP FOUR

Computing the gender-related development index (GDI)

$1/3(0.875 + 0.968 + 0.959) = 0.934$

The gender empowerment measure

The GEM uses variables constructed explicitly to measure the relative empowerment of women and men in political and economic spheres of activity.

The first two variables are chosen to reflect economic participation and decision-making power: women's and men's percentage shares of administrative and managerial positions and their percentage shares of professional and technical jobs. These are broad, loosely defined occupational categories. Because the relevant population for each is different, we calculate a seperate index for each and then add the two together. The third variable, women's and men's percentage shares of parliamentary seats, is chosen to reflect political participation and decision-making power.

For all three variables we use the methodology of population-weighted $(1 - \in)$ averaging to derive an "equally distributed equivalent percentage" (EDEP) for both sexes taken together. Each variable is indexed by dividing the EDEP by 50%.

An income variable is used to reflect power over economic resources. It is calculated in the same manner as for the GDI except that unadjusted rather than adjusted real GDP per capita is used. The maximum value for income is thus PPP$40,000 and the minimum PPP$100.

The three indices—for economic participation and decision-making, political participation and decision-making, and power over economic resources—are added together to derive the final GEM value.

Illustration of the GEM methodology

We choose Cameroon to illustrate the steps in calculating the GEM. The parameter of inequality aversion, $\in$, equals 2. (Any discrepancies in results are due to rounding.)

STEP ONE

Computing indices for parliamentary representation and administrative and managerial, and professional and technical, positions

Percentage share of parliamentary representation	
Females	12.1
Males	87.8
Percentage share of administrative and managerial positions	
Females	10.1
Males	89.9
Percentage share of professional and technical positions	
Females	24.4
Males	75.6
Percentage share of population	
Females	50.38
Males	49.62

Computing the EDEP for parliamentary representation

$[0.4962(87.8)^{-1} + 0.5038(12.1)^{-1}] = 21.3$

Computing the EDEP for administrative and managerial positions

$[0.4962(89.9)^{-1} + 0.5038(10.1)^{-1}] = 18.05$

Computing the EDEP for professional and technical positions

$[0.4962(75.6)^{-1} + 0.5038(24.4)^{-1}] = 36.75$

Indexing parliamentary representation

$21.30/50 = 0.426$

Indexing administrative and managerial positions

$18.05/50 = 0.361$

Indexing professional and technical positions

$36.75/50 = 0.735$

Combining the indices for administrative and managerial, and professional and technical, positions

$(0.361 + 0.735)/2 = 0.548$

STEP TWO

Computing the index for share of earned income

Percentage share of economically active population	
Females	37.4
Males	62.6

Ratio of female non-agricultural wage to male non-agricultural wage: 75%

Unadjusted real GDP per capita: PPP$2,120

Ratio of female wage to average wage (W) and of male wage to average wage (W):

$W = 0.374(0.75) + 0.626(1) = 0.9065$

Female wage to average wage: $0.75/0.9065 = 0.8274$

Male wage to average wage: $1/0.9065 = 1.1031$

Share of earned income

Note: [(female wage/average wage) × female share of economically active population] + [(male wage/average wage) × male share of economically active population] = 1

Females	$0.8274 \times 0.374 = 0.3094$
Males	$1.1031 \times 0.626 = 0.6095$

Female and male proportional income shares

Females	$0.3094/0.5038 = 0.6141$
Males	$0.6905/0.4962 = 1.3916$

Computing the equally distributed income index

$[0.4962(1.3916)^{-1} + 0.5038(0.6141)^{-1}]^{-1} = 0.8496$

$0.8496 \times 2{,}120 = 1{,}801$

$(1{,}801 - 100)/(40{,}000 - 100) = 0.0426$

STEP THREE

Computing the GEM

$[1/3(0.0426 + 0.0548 + 0.426)] = 0.3389$

The human poverty index

The HPI concentrates on deprivation in three essential elements of human life already reflected in the HDI—longevity, knowledge and a decent standard of living. The first deprivation relates to survival—the vulnerability to death at a relatively early age. The second relates to knowledge—being excluded from the world of reading and communication. The third relates to a decent living standard in terms of overall economic provisioning.

In constructing the HPI, the deprivation in longevity is represented by the percentage of people not expected to survive to age 40 (P_1), and the deprivation in knowledge by the percentage of adults who are illiterate (P_2). The deprivation in a decent living standard in terms of overall economic provisioning is represented by a composite (P_3) of three variables—the percentage of people without access to safe water (P_{31}), the percentage of people without access to health services (P_{32}) and the percentage of moderately and severely underweight children under five (P_{33}).

The composite variable P is constructed by taking a simple average of the three variables P_{31}, P_{32} and P_{33}. Thus

$$P_3 = \frac{P_{31} + P_{32} + P_{33}}{3} \; .$$

Following the analysis in chapter 1 and technical note 1, the formula of HPI is given by

$$\text{HPI} = [(P_1^3 + P_2^3 + P_3^3) - 3]^{1/3}.$$

As an example, we compute the HPI for Egypt.

STEP ONE
Calculating P_3

Country	P_1 (%)	P_2 (%)	P_{31} (%)	P_{32} (%)	P_{33} (%)
Egypt	16.6	49.5	21.0	1.0	9.0

$$P_3 = \frac{21 + 1 + 9}{3} = \frac{31}{3} = 10.33$$

STEP TWO
Constructing the HPI

$$\begin{aligned}\text{HPI} &= [1/3(16.6^3 + 49.5^3 + 10.33^3)]^{1/3} \\ &= [1/3(4,574.30 + 121,287.38 + 1,102.30)]^{1/3} \\ &= [1/3(126,963.98)^{1/3} \\ &= (42,321.33)^{1/3} \\ &= 34.8\end{aligned}$$

TECHNICAL NOTE TABLE 2.1
Human poverty index

HPI rank		Survival deprivation People not expected to survive to age 40 (%) 1990[a] (P_1)	Deprivation in education and knowledge Adult illiteracy rate (%) 1994 (P_2)	Deprivation in economic provisioning (P_3) Population without access to safe water (%) 1990–96 (P_{31})	Population without access to health services (%) 1990–95 (P_{32})	Underweight children under age five (%) 1990–96 (P_{33})	Overall (P_3)	Human poverty index (HPI) value (%)
1	Trinidad and Tobago	5.4[b]	2.1	3	0	7[c]	3	4.1
2	Cuba	6.2[d,e]	4.6	11	0	1[f]	4	5.1
3	Chile	4.6[d,e]	5.0	15[c]	3[c]	1	6	5.4
4	Singapore	3.2[d,e]	9.0	0[c]	0[c]	14[c]	5	6.6
5	Costa Rica	4.1[b]	5.3	4	20[c]	2	9	6.6
6	Colombia	6.3[b]	8.9	15	19	8	14	10.7
7	Mexico	8.3[b]	10.8	17	7	14[c]	13	10.9
8	Jordan	9.2[b]	14.5	2	3[c]	9	5	10.9
9	Panama	6.2[d,e]	9.5	7	30	7	15	11.2
10	Uruguay	5.4[d,e]	2.9	25[c]	18[c]	7[c]	17	11.7
11	Thailand	8.9[b]	6.5	11	10[c]	26[c]	16	11.7
12	Jamaica	4.3[b]	15.6	14	10[c]	10	11	12.1
13	Mauritius	6.2[d,e]	17.6	1	0[c]	16	6	12.5
14	United Arab Emirates	3.6[b]	21.4	5	1	6[g]	4	14.9
15	Ecuador	9.9[b]	10.4	32	12[c]	17[c]	20	15.2
16	Mongolia	16.0[h,i]	17.8	20	5[c]	12	12	15.7
17	Zimbabwe	18.4[d,j]	15.3	23	15	16	18	17.3
18	China	9.1[d,k]	19.1	33	12	16	20	17.5
19	Philippines	12.8[d,l]	5.6	14	29	30	24	17.7
20	Dominican Rep.	10.2[b]	18.5	35	22	10	22	18.3
21	Libyan Arab Jamahiriya	16.2[b]	25.0	3	5	5	4	18.8
22	Sri Lanka	7.9[d,e]	9.9	43	7[c]	38	29	20.7
23	Indonesia	14.8[d,j]	16.8	38	7	35	27	20.8
24	Syrian Arab Rep.	10.3[b]	30.2	15	10	12	12	21.7
25	Honduras	10.8[b]	28.0	13	31	18	21	22.0
26	Bolivia	19.6[d,j]	17.5	34	33	16	28	22.5
27	Iran, Islamic Rep. of	11.7[b]	31.4[m]	10	12	16	13	22.6
28	Peru	13.4[d,j]	11.7	28	56	11	32	22.8
29	Botswana	15.9[b]	31.3	7[c]	11[c]	15[c]	11	22.9
30	Paraguay	9.2[b]	8.1	58	37[c]	4	33	23.2
31	Tunisia	10.5[b]	34.8	2	10[c]	9	7	24.4
32	Kenya	22.3[b]	23.0	47	23	23	31	26.1
33	Viet Nam	12.1[b]	7.0	57	10	45	37	26.2
34	Nicaragua	13.6[b]	34.7	47	17[c]	12	25	27.2
35	Lesotho	23.9[b]	29.5	44	20[c]	21	28	27.5
36	El Salvador	11.7[b]	29.1	31	60	11	34	28.0
37	Algeria	10.6[b]	40.6	22	2	13	12	28.6
38	Congo	22.1[b]	26.1	66	17[c]	24[c]	36	29.1
39	Iraq	15.4[b]	43.2	22	7[c]	12	14	30.7
40	Myanmar	25.6[b]	17.3	40	40	43	41	31.2
41	Cameroon	25.4[b]	37.9	50	20	14	28	31.4
42	Papua New Guinea	28.6[b]	28.8	72	4[c]	35[c]	37	32.0
43	Ghana	24.9[b]	36.6	35	40[c]	27	34	32.6
44	Egypt	16.6[de]	49.5	21	1	9	10	34.8
45	Zambia	35.1[b]	23.4	73	25[c]	28	42	35.1
46	Guatemala	14.5[d,e]	44.3	36	43	27	35	35.5
47	India	19.4[d,k]	48.8	19	15	53	29	36.7
48	Rwanda	42.1[b]	40.8	34[c]	20	29	28	37.9
49	Togo	28.4[b]	49.6	37	39[c]	24[c]	33	39.3
50	Tanzania, U. Rep. of	30.6[b]	33.2	62	58	29	50	39.7
51	Lao People's Dem. Rep.	32.7[h,i]	44.2	48	33[c]	44	42	40.1
52	Zaire	30.0[b]	23.6	58	74[c]	34	55	41.2
53	Uganda	39.0[d,n]	38.9	62	51	23[c]	45	41.3
54	Nigeria	33.8[b]	44.4	49	49	36	45	41.6
55	Morocco	12.3[d,l]	57.9	45	30[c]	9	28	41.7

TECHNICAL NOTE TABLE 2.1

Human poverty index (continued)

HPI rank	Survival deprivation People not expected to survive to age 40 (%) 1990[a] (P_1)	Deprivation in education and knowledge Adult illiteracy rate (%) 1994 (P_2)	Deprivation in economic provisioning (P_3) Population without access to safe water (%) 1990–96 (P_{31})	Population without access to health services (%) 1990–95 (P_{32})	Underweight children under age five (%) 1990–96 (P_{33})	Overall (P_3)	Human poverty index (HPI) value (%)
56 Central African Rep.	35.4[d,j]	42.8	62	48	27	46	41.7
57 Sudan	25.2[b]	55.2	40	30	34	35	42.2
58 Guinea-Bissau	43.2[h,i]	46.1	41	60	23[c]	41	43.6
59 Namibia	21.1[d,j]	60.0[o]	43	41	26	37	45.1
60 Malawi	38.3[d,j]	44.2	63	65	30	53	45.8
61 Haiti	27.1[b]	55.9	72	40	28	47	46.2
62 Bhutan	33.2[h,i]	58.9	42	35[c]	38[c]	38	46.3
63 Côte d'Ivoire	23.1[d,n]	60.6	25	70[c]	24	40	46.3
64 Pakistan	22.6[b]	62.9	26	45[c]	38	36	46.8
65 Mauritania	31.7[h,i]	63.1	34[c]	37	23	31	47.1
66 Yemen	25.6[b]	58.9[o]	39	62	39	47	47.6
67 Bangladesh	26.4[b]	62.7	3	55	67	42	48.3
68 Senegal	25.3[d,p]	67.9	48	10	20	26	48.7
69 Burundi	33.8[b]	65.4	41	20	37	33	49.0
70 Madagascar	32.1[d,i]	54.2[q]	71	62	34	56	49.5
71 Guinea	41.3[h,i]	65.2	45	20	26	30	50.0
72 Mozambique	43.8[b]	60.5	37	61[c]	27	42	50.1
73 Cambodia	31.9[h,i]	65.0[r]	64	47[c]	40	50	52.5
74 Mali	28.4[d,n]	70.7	55	60	31[c]	49	54.7
75 Ethiopia	35.7[b]	65.5	75	54	48	59	56.2
76 Burkina Faso	36.1[b]	81.3	22	10	30	21	58.3
77 Sierra Leone	52.1[b]	69.7	66	62	29	52	59.2
78 Niger	43.2[d]	86.9	46	68[r]	36	50	66.0

a. Data refer to 1990 or a year around 1990. b. Obtained by combining two sets of mortality risk estimates: UNICEF estimates of the probability of dying by age 5 and UN Population Division estimates of the probability of dying between the ages of 5 and 40 ($_{35}q_5$). Estimates were interpolated using the Coale-Demeny "West" family of model life tables. For all countries life expectancy at birth in 1990 is estimated as the arithmetic average of estimates for that period in UN 1996b, as explained in Hill 1997. c. Data refer to a year or period other than that specified in the column heading, differ from the standard definition or refer to only part of a country. d. UNICEF estimates of the probability of dying by age 5 plus independent estimates (Hill 1997) of the probability of dying between the ages of 5 and 40. e. Based on registration of deaths around 1990. f. Wasting (moderate or severe). g. UNICEF field office source. h. UN Population Division, based on life expectancy at birth. i. UN Population Division, obtained by finding estimated life expectancy at birth in 1990 (obtained by linear interpolation between the 1985–90 and 1990–95 estimates) and then finding the implied $_{40}q_0$ and $_{60}q_0$ values in the Coale-Demeny "West" model life tables. Keyfitz and Flieger national life tables were used to calculate the ratio of life expectancy at birth to $_{40}q_0$ and $_{60}q_0$ around 1970 and around 1985; for each country the ratios for 1990 were then estimated by linear extrapolation. These ratios were plotted against time and found to change in similar ways over time across countries, giving a series of parallel lines. The estimated ratio and estimated life expectancy at birth were then used to obtain estimated risks of dying by age 40 and age 60, as explained in Hill 1997. j. Based on Demographic and Health Survey direct sisterhood estimates of probability of dying between the ages of 5 and 50, extended from 50 to 60 using a Coale-Demeny "West" model life table fitted to $_{45}q_5$, as explained in Hill 1997. k. World Bank 1993. l. Based on registered deaths after adjusting for estimated completeness. m. UNESCO 1995. Data are for 1995. n. Based on Demographic and Health Survey direct sisterhood estimates of probability of dying between the ages of 15 and 50, extended to cover ages 5 to 14 and 50 to 60 using a Coale-Demeny "West" model life table fitted to $_{35}q_{15}$. o. UNDP 1996d. p. Pison and others 1995. q. Human Development Report Office estimate based on national sources. r. UNICEF 1996b.

Source: Column 1: Hill 1997; *column 2:* calculated on the basis of data from UNESCO 1996b; *columns 3 and 4:* calculated on the basis of data from UNICEF 1997; *column 5:* UNICEF 1997.