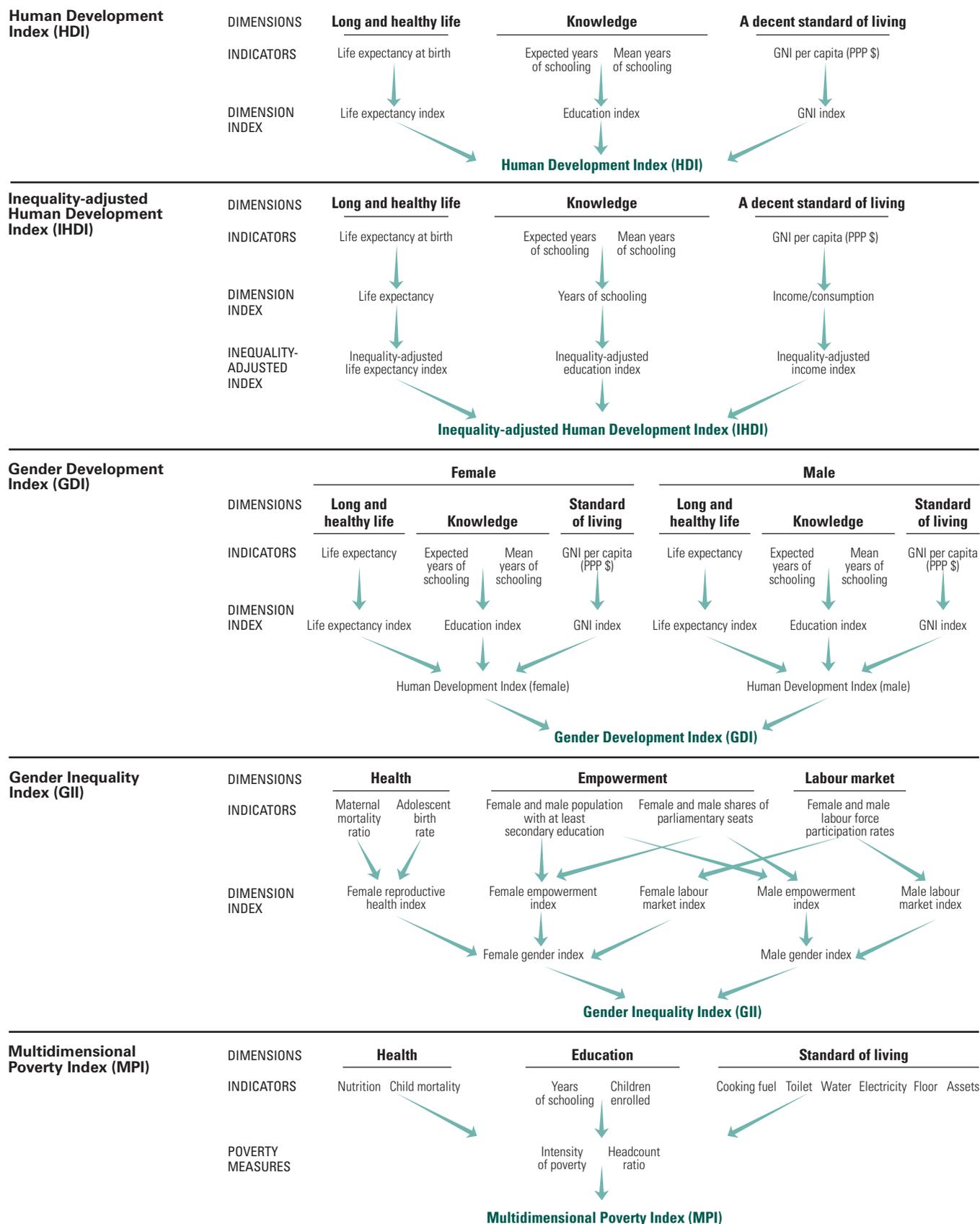


# Technical notes

## Calculating the human development indices—graphical presentation



## Technical note 1. Human Development Index

The Human Development Index (HDI) is a summary measure of achievements in three key dimensions of human development: a long and healthy life, access to knowledge and a decent standard of living. The HDI is the geometric mean of normalized indices for each of the three dimensions.

### Data sources

- Life expectancy at birth: UNDESA (2015).
- Expected years of schooling: UNESCO Institute for Statistics (2016), United Nations Children’s Fund (UNICEF) Multiple Indicator Cluster Surveys and ICF Macro Demographic and Health Surveys.
- Mean years of schooling: Barro and Lee (2016), UNESCO Institute for Statistics (2016), Human Development Report Office updates based on UNESCO Institute for Statistics (2016), UNICEF Multiple Indicator Cluster Surveys and ICF Macro Demographic and Health Surveys.
- GNI per capita: IMF (2016), UNSD (2016) and World Bank (2016).

### Steps to calculate the Human Development Index

There are two steps to calculating the HDI.

#### Step 1. Creating the dimension indices

Minimum and maximum values (goalposts) are set in order to transform the indicators expressed in different units into indices on a scale of 0 to 1. These goalposts act as the “natural zeros” and “aspirational targets,” respectively, from which component indicators are standardized (see equation 1 below). They are set at the following values:

Dimension	Indicator	Minimum	Maximum
Health	Life expectancy (years)	20	85
Education	Expected years of schooling (years)	0	18
	Mean years of schooling (years)	0	15
Standard of living	Gross national income per capita (2011 PPP \$)	100	75,000

The justification for placing the natural zero for life expectancy at 20 years is based on historical evidence that no country in the 20th century had a life expectancy of less than 20 years (Maddison 2010; Oeppen and Vaupel 2002; Riley 2005).

Societies can subsist without formal education, justifying the education minimum of 0 years. The maximum for expected years of schooling, 18, is equivalent to achieving a master’s degree in

most countries. The maximum for mean years of schooling, 15, is the projected maximum of this indicator for 2025.

The low minimum value for gross national income (GNI) per capita, \$100, is justified by the considerable amount of unmeasured subsistence and nonmarket production in economies close to the minimum, which is not captured in the official data. The maximum is set at \$75,000 per capita. Kahneman and Deaton (2010) have shown that there is virtually no gain in human development and well-being from income per capita above \$75,000. Currently, only four countries (Kuwait, Liechtenstein, Qatar and Singapore) exceed the \$75,000 income per capita ceiling.

Having defined the minimum and maximum values, the dimension indices are calculated as:

$$\text{Dimension index} = \frac{\text{actual value} - \text{minimum value}}{\text{maximum value} - \text{minimum value}} \quad (1)$$

For the education dimension, equation 1 is first applied to each of the two indicators, and then the arithmetic mean of the two resulting indices is taken.

Because each dimension index is a proxy for capabilities in the corresponding dimension, the transformation function from income to capabilities is likely to be concave (Anand and Sen 2000)—that is, each additional dollar of income has a smaller effect on expanding capabilities. Thus for income the natural logarithm of the actual, minimum and maximum values is used.

#### Step 2. Aggregating the dimensional indices to produce the Human Development Index

The HDI is the geometric mean of the three dimension indices:

$$HDI = (I_{Health} \cdot I_{Education} \cdot I_{Income})^{1/3}$$

#### Example: Georgia

Indicator	Value
Life expectancy at birth (years)	75.0
Expected years of schooling (years)	13.9
Mean years of schooling (years)	12.2
Gross national income per capita (2011 PPP \$)	8,856

Note: Values are rounded.

$$\text{Health index} = \frac{75.020 - 20}{85 - 20} = 0.8465$$

$$\text{Expected years of schooling index} = \frac{13.905 - 0}{18 - 0} = 0.77249$$

$$\text{Mean years of schooling index} = \frac{12.246 - 0}{15 - 0} = 0.81643$$

$$\text{Education index} = \frac{0.81643 + 0.77249}{2} = 0.7945$$

$$\text{Income index} = \frac{\ln(8,855.8) - \ln(100)}{\ln(75,000) - \ln(100)} = 0.6773$$

$$\text{Human Development Index} = (0.8465 \cdot 0.7945 \cdot 0.6773)^{\frac{1}{3}} = 0.769$$

## Methodology used to express income

The World Bank’s 2016 World Development Indicators database contains estimates of GNI per capita in constant 2011 purchasing power parity (PPP) terms for many countries. For countries missing this indicator (entirely or partly), the Human Development Report Office calculates it by converting GNI per capita from current to constant terms using two steps. First, the value of GNI per capita in current terms is converted into PPP terms for the base year (2011). Second, a time series of GNI per capita in constant 2011 PPP terms is constructed by applying the real growth rates to the GNI per capita in PPP terms for the base year. The real growth rate is implied by the ratio of the nominal growth of current GNI per capita in local currency terms to the GDP deflator.

To obtain the income value for 2016 for some countries, the International Monetary Fund (IMF)–projected real growth rates of GDP are applied to the most recent GNI values in constant PPP terms. The IMF-projected real growth rates are calculated based on local currency terms and constant prices rather than in PPP terms. This avoids mixing the effects of the PPP conversion with those of real growth of the economy.

## Technical note 2. Inequality-adjusted Human Development Index

The Inequality-adjusted Human Development Index (IHDI) adjusts the Human Development Index (HDI) for inequality in the distribution of each dimension across the population. It is based on a distribution-sensitive class of composite indices proposed by Foster, Lopez-Calva and Szekely (2005), which draws on the Atkinson (1970) family of inequality measures. It is computed as a geometric mean of inequality-adjusted dimensional indices.

The IHDI accounts for inequalities in HDI dimensions by “discounting” each dimension’s average value according to its level of inequality. The IHDI equals the HDI when there is no inequality across people but falls below the HDI as inequality

Official PPP conversion rates are produced by the International Comparison Program, whose surveys periodically collect thousands of prices of matched goods and services in many countries. The last round of this exercise refers to 2011 and covered 199 countries.

## Estimating missing values

For a small number of countries missing one of the four indicators, the Human Development Report Office estimated the missing values using cross-country regression models. The details of the models used are available at <http://hdr.undp.org>.

In this Report expected years of schooling were estimated for the Bahamas, Bahrain, Dominica, Equatorial Guinea, the Federated States of Micronesia, Fiji, Haiti, Iraq, Papua New Guinea and the United Arab Emirates, and mean years of schooling were estimated for Antigua and Barbuda, Cabo Verde, Eritrea, Grenada, Guinea-Bissau, Kiribati, Palau, Saint Kitts and Nevis, Saint Vincent and the Grenadines, Seychelles and Turkmenistan.

## Country groupings

This Report keeps the same cutoff points of the HDI for grouping countries that were introduced in the 2014 Report:

Very high human development	0.800 and above
High human development	0.700–0.799
Medium human development	0.550–0.699
Low human development	Below 0.550

rises. In this sense, the IHDI measures the level of human development when inequality is accounted for.

## Data sources

Since the HDI relies on country-level aggregates such as national accounts for income, the IHDI must draw on additional sources of data to obtain insights into the distribution. The distributions are observed over different units—life expectancy is distributed across a hypothetical cohort, while years of schooling and income are distributed across individuals.

Inequality in the distribution of HDI dimensions is estimated for:

- Life expectancy, using data from abridged life tables provided by UNDESA (2015). This distribution is presented over age intervals (0–1, 1–5, 5–10, ... , 85+), with the mortality rates and average age at death specified for each interval.
- Mean years of schooling, using household surveys data harmonized in international databases, including the Luxembourg Income Study, Eurostat’s European Union Survey of Income and Living Conditions, the World Bank’s International Income Distribution Database, United Nations Children’s Fund Multiple Indicator Cluster Surveys, ICF Macro Demographic and Health Surveys and the United Nations University’s World Income Inequality Database.
- Disposable household income or consumption per capita using the above listed databases and household surveys—and for a few countries, income imputed based on an asset index matching methodology using household survey asset indices (Harttgen and Vollmer 2013).

A full account of data sources used for estimating inequality in 2015 is available at <http://hdr.undp.org/en/statistics/ihdi/>.

## Steps to calculate the Inequality-adjusted Human Development Index

There are three steps to calculating the IHDI.

### Step 1. Estimating inequality in the dimensions of the Human Development Index

The IHDI draws on the Atkinson (1970) family of inequality measures and sets the aversion parameter  $\epsilon$  equal to 1.<sup>1</sup> In this case the inequality measure is  $A = 1 - g/\mu$ , where  $g$  is the geometric mean and  $\mu$  is the arithmetic mean of the distribution. This can be written as:

$$A_x = 1 - \frac{\sqrt[n]{X_1 \dots X_n}}{\bar{X}} \quad (1)$$

where  $\{X_1, \dots, X_n\}$  denotes the underlying distribution in the dimensions of interest.  $A_x$  is obtained for each variable (life expectancy, mean years of schooling and disposable household income or consumption per capita).

The geometric mean in equation 1 does not allow zero values. For mean years of schooling one year is added to all valid observations to compute the inequality. Income per capita outliers—extremely high incomes as well as negative and zero incomes—were dealt with by truncating the top 0.5 percentile of the distribution to reduce the influence of extremely high

incomes and by replacing the negative and zero incomes with the minimum value of the bottom 0.5 percentile of the distribution of positive incomes. Sensitivity analysis of the IHDI is given in Kovacevic (2010).

### Step 2. Adjusting the dimension indices for inequality

The inequality-adjusted dimension indices are obtained from the HDI dimension indices,  $I_x$ , by multiplying them by  $(1 - A_x)$ , where  $A_x$ , defined by equation 1, is the corresponding Atkinson measure:

$$I_x^* = (1 - A_x) \cdot I_x.$$

The inequality-adjusted income index,  $I_{income}^*$ , is based on the index of logged income values,  $I_{income}$ , and inequality in income distribution computed using income in levels. This enables the IHDI to account for the full effect of income inequality.

### Step 3. Combining the dimension indices to calculate the Inequality-adjusted Human Development Index

The IHDI is the geometric mean of the three dimension indices adjusted for inequality:

$$IHDI = (I_{Health}^* \cdot I_{Education}^* \cdot I_{Income}^*)^{1/3} = [(1 - A_{Health}) \cdot (1 - A_{Education}) \cdot (1 - A_{Income})]^{1/3} \cdot HDI.$$

The loss in the Human Development Index due to inequality is:

$$Loss = 1 - [(1 - A_{Health}) \cdot (1 - A_{Education}) \cdot (1 - A_{Income})]^{1/3}.$$

## Coefficient of human inequality

An unweighted average of inequalities in health, education and income is denoted as the coefficient of human inequality. It averages these inequalities using the arithmetic mean:

$$Coefficient\ of\ human\ inequality = \frac{A_{Health} + A_{Education} + A_{Income}}{3}.$$

When all inequalities in dimensions are of a similar magnitude the coefficient of human inequality and the loss in HDI differ negligibly. When inequalities differ in magnitude, the loss in HDI tends to be higher than the coefficient of human inequality.

## Notes on methodology and caveats

The IHDI is based on the Atkinson index, which satisfies subgroup consistency. This property ensures that improvements (deteriorations) in the distribution of human development within only a certain group of the society imply improvements (deteriorations) in the distribution across the entire society.

The main disadvantage is that the IHDI is not association sensitive, so it does not capture overlapping inequalities. To make the measure association sensitive, all the data for each individual must be available from a single survey source, which is not currently possible for a large number of countries.

### Example: Bolivia (Plurinational State of)

Indicator	Value	Dimension index	Inequality measure <sup>a</sup> (A)	Inequality-adjusted index (I*)
Life expectancy (years)	68.7	0.7499	0.290	(1-0.2905) · 0.7499 = 0.5321
Expected years of schooling (years)	13.8	0.7600	—	—
Mean years of schooling (years)	8.2	0.5466	0.208	—
Education index	—	0.6563	0.208	(1-0.2078) · 0.6563 = 0.5199
Gross national income per capita (2011 PPP \$)	6,155	0.6223	0.364	(1-0.3644) · 0.6223 = 0.3955
<b>Human Development Index</b>		<b>Inequality-adjusted Human Development Index</b>		
(0.7499 · 0.6563 · 0.6223) <sup>1/3</sup> = 0.674		(0.5321 · 0.5199 · 0.3955) <sup>1/3</sup> = 0.478		
<b>Loss due to inequality (%)</b>		<b>Coefficient of human inequality (%)</b>		
100 · (1 - $\frac{0.478}{0.674}$ ) = 29.0		$\frac{100 \cdot (0.290 + 0.208 + 0.364)}{3} = 28.7$		

Note: Values are rounded.  
a. Inequalities are estimated from micro data.

## Technical note 3. Gender Development Index

The Gender Development Index (GDI) measures gender inequalities in achievement in three basic dimensions of human development: health, measured by female and male life expectancy at birth; education, measured by female and male expected years of schooling for children and female and male mean years of schooling for adults ages 25 years and older; and command over economic resources, measured by female and male estimated earned income.

### Data sources

- Life expectancy at birth: UNDESA (2015).
- Expected years of schooling: UNESCO Institute for Statistics (2016), United Nations Children's Fund (UNICEF) Multiple Indicator Cluster Surveys and ICF Macro Demographic and Health Surveys.
- Mean years of schooling for adults ages 25 and older: Barro and Lee (2016), UNESCO Institute for Statistics (2016), UNICEF Multiple Indicator Cluster Surveys and ICF Macro Demographic and Health Surveys.
- Estimated earned income: Human Development Report Office estimates based on female and male shares of the economically active population, the ratio of the female to male wage in all sectors and gross national income in 2011 purchasing power parity (PPP) terms, and female and male shares of population from ILO (2016), IMF (2016), UNDESA (2015) and World Bank (2016).

### Steps to calculate the Gender Development Index

There are four steps to calculating the GDI.

#### Step 1. Estimating the female and male earned incomes

To calculate estimated earned incomes, the share of the wage bill is calculated for each gender. The female share of the wage bill ( $S_f$ ) is calculated as follows:

$$S_f = \frac{W_f/W_m \cdot EA_f}{W_f/W_m \cdot EA_f + EA_m}$$

where  $W_f/W_m$  is the ratio of female to male wage,  $EA_f$  is the female share of the economically active population and  $EA_m$  is the male share of the economically active population.

The male share of the wage bill is calculated as:

$$S_m = 1 - S_f$$

Estimated female earned income per capita ( $GNIpc_f$ ) is obtained from GNI per capita ( $GNIpc$ ), first by multiplying it by the female share of the wage bill,  $S_f$ , and then rescaling it by the female share of the population,  $P_f = N_f/N$ :

$$GNIpc_f = GNIpc \cdot S_f / P_f$$

Estimated male earned income per capita is obtained in the same way:

$$GNIpc_m = GNIpc \cdot S_m / P_m$$

## Step 2. Normalizing the indicators

To construct the female and male HDI values, first the indicators, which are in different units, are transformed into indices and then dimension indices for each sex are aggregated by taking the geometric mean.

The indicators are transformed into indices on a scale of 0 to 1 using the same goalposts that are used for the HDI, except life expectancy at birth, which is adjusted for the average five-year biological advantage that women have over men.

### Goalposts for the Gender Development Index in this Report

Indicator	Minimum	Maximum
Expected years of schooling (years)	0	18
Mean years of schooling (years)	0	15
Estimated earned income (2011 PPP \$)	100	75,000
Life expectancy at birth (years)		
Female	22.5	87.5
Male	17.5	82.5

Note: For rationale on choice of minimum and maximum values, see *Technical note 1*.

Having defined the minimum and maximum values, the subindices are calculated as follows:

$$\text{Dimension index} = \frac{\text{actual value} - \text{minimum value}}{\text{maximum value} - \text{minimum value}}$$

For education the dimension index is first obtained for each of the two subcomponents, and then the unweighted arithmetic mean of the two resulting indices is taken.

## Step 3. Calculating the female and male Human Development Index values

The female and male HDI values are the geometric means of the three dimensional indices for each gender:

$$HDI_f = (I_{Health_f} \cdot I_{Education_f} \cdot I_{Income_f})^{1/3}$$

$$HDI_m = (I_{Health_m} \cdot I_{Education_m} \cdot I_{Income_m})^{1/3}$$

## Step 4: Calculating the Gender Development Index

The GDI is simply the ratio of female HDI to male HDI:

$$GDI = \frac{HDI_f}{HDI_m}$$

### Example: Malawi

Indicator	Female value	Male value
Life expectancy at birth (years)	64.8	62.9
Expected years of schooling (years)	10.7	10.8
Mean years of schooling (years)	3.8	5.0
Wage ratio (female/male)	0.8 (imputed)	
Gross national income per capita (2011 PPP \$)	1,073.29	
Share of economically active population	0.509	0.491
Share of population	0.5008	0.4992

#### Female wage bill:

$$S_f = (0.8 \cdot 0.509) / [(0.8 \cdot 0.509) + 0.491] = 0.4534$$

#### Estimated female earned income per capita:

$$GNIpc_f = 1,073.29 \cdot 0.4534 / 0.5008 = 971.7$$

#### Male wage bill:

$$S_m = 1 - 0.4534 = 0.5466$$

#### Estimated male earned income per capita:

$$GNIpc_m = 1,073.29 \cdot 0.5466 / 0.4992 = 1,175.2$$

$$\text{Female health index} = (64.8 - 22.5) / (87.5 - 22.5) = 0.6508$$

$$\text{Male health index} = (62.9 - 17.5) / (82.5 - 17.5) = 0.6985$$

$$\text{Female education index} = [(10.7 / 18) + (3.8 / 15)] / 2 = 0.4239$$

$$\text{Male education index} = [(10.8 / 18) + (5.0 / 15)] / 2 = 0.4667$$

#### Estimated female earned income index:

$$[\ln(971.7) - \ln(100)] / [(\ln(75,000) - \ln(100))] = 0.3435$$

#### Estimated male earned income index:

$$[\ln(1,175.2) - \ln(100)] / [(\ln(75,000) - \ln(100))] = 0.3722$$

$$\text{Female HDI} = (0.6508 \cdot 0.4239 \cdot 0.3435)^{1/3} = 0.455$$

$$\text{Male HDI} = (0.6985 \cdot 0.4667 \cdot 0.3722)^{1/3} = 0.495$$

$$GDI = 0.455 / 0.495 = 0.921$$

Note: Values are rounded.

### GDI groups

The GDI groups are based on the absolute deviation of GDI from gender parity,  $100 \cdot |GDI - 1|$ . Countries with absolute deviation from gender parity of 2.5 percent or less are considered countries with high equality in HDI achievements between women and men and are classified as group 1. Countries with absolute deviation from gender parity of 2.5–5 percent are considered

countries with medium-high equality in HDI achievements between women and men and are classified as group 2. Countries with absolute deviation from gender parity of 5–7.5 percent are considered countries with medium equality in HDI achievements between women and men and are classified as group 3. Countries with absolute deviation from gender parity of 7.5–10

percent are considered countries with medium-low equality in HDI achievements between women and men and are classified as group 4. Countries with absolute deviation from gender parity of more than 10 percent are considered countries with low equality in HDI achievements between women and men and are classified as group 5.

## Technical note 4. Gender Inequality Index

The Gender Inequality Index (GII) reflects gender-based disadvantage in three dimensions—reproductive health, empowerment and the labour market—for as many countries as data of reasonable quality allow. It shows the loss in potential human development due to inequality between female and male achievements in these dimensions. It ranges from 0, where women and men fare equally, to 1, where one gender fares as poorly as possible in all measured dimensions.

The GII is computed using the association-sensitive inequality measure suggested by Seth (2009), which implies that the index is based on the general mean of general means of different orders—the first aggregation is by a geometric mean across dimensions; these means, calculated separately for women and men, are then aggregated using a harmonic mean across genders.

### Data sources

- Maternal mortality ratio (*MMR*): UN Maternal Mortality Estimation Group (2016).
- Adolescent birth rate (*ABR*): UNDESA (2015).
- Share of parliamentary seats held by each sex (*PR*): IPU (2016).
- Attainment at secondary and higher education levels (*SE*): UNESCO Institute for Statistics (2016).
- Labour market participation rate (*LFPR*): ILO (2016).

### Steps to calculate the Gender Inequality Index

There are five steps to calculating the GII.

#### Step 1. Treating zeros and extreme values

Because a geometric mean cannot be computed from zero values, a minimum value of 0.1 percent is set for all component indicators. Further, as higher maternal mortality suggests poorer maternal health, for the maternal mortality ratio the maximum value is truncated at 1,000 deaths per 100,000 births

and the minimum value at 10. The rationale is that countries where maternal mortality ratios exceed 1,000 do not differ in their inability to create conditions and support for maternal health and that countries with 10 or fewer deaths per 100,000 births are performing at essentially the same level and that small differences are random.

Sensitivity analysis of the GII is given in Gaye et al. (2010).

#### Step 2. Aggregating across dimensions within each gender group, using geometric means

Aggregating across dimensions for each gender group by the geometric mean makes the GII association sensitive (see Seth 2009).

For women and girls, the aggregation formula is:

$$G_F = \sqrt[3]{\left(\frac{10}{MMR} \cdot \frac{1}{ABR}\right)^{1/2} \cdot (PR_F \cdot SE_F)^{1/2} \cdot LFPR_F}, \quad (1)$$

and for men and boys the formula is

$$G_M = \sqrt[3]{1 \cdot (PR_M \cdot SE_M)^{1/2} \cdot LFPR_M}.$$

The rescaling by 0.1 of the maternal mortality ratio in equation 1 is needed to account for the truncation of the maternal mortality ratio at 10.

#### Step 3. Aggregating across gender groups, using a harmonic mean

The female and male indices are aggregated by the harmonic mean to create the equally distributed gender index

$$HARM(G_F, G_M) = \left[ \frac{(G_F)^{-1} + (G_M)^{-1}}{2} \right]^{-1}.$$

Using the harmonic mean of geometric means within groups captures the inequality between women and men and adjusts for association between dimensions—that is, it accounts for the overlapping inequalities in dimensions.

#### Step 4. Calculating the geometric mean of the arithmetic means for each indicator

The reference standard for computing inequality is obtained by aggregating female and male indices using equal weights (thus treating the genders equally) and then aggregating the indices across dimensions:

$$G_{F,M}^- = \sqrt[3]{\overline{Health} \cdot \overline{Empowerment} \cdot \overline{LFPR}}$$

$$\text{where } \overline{Health} = \left( \sqrt{\frac{10}{MMR} \cdot \frac{1}{ABR} + 1} \right) / 2,$$

$$\overline{Empowerment} = \left( \sqrt{PR_F \cdot SE_F} + \sqrt{PR_M \cdot SE_M} \right) / 2 \text{ and}$$

$$\overline{LFPR} = \frac{LFPR_F + LFPR_M}{2}.$$

$\overline{Health}$  should not be interpreted as an average of corresponding female and male indices but rather as half the distance from the norms established for the reproductive health indicators—fewer maternal deaths and fewer adolescent pregnancies.

#### Step 5. Calculating the Gender Inequality Index

Comparing the equally distributed gender index to the reference standard yields the GII,

$$1 - \frac{HARM(G_F, G_M)}{G_{F,M}^-}.$$

### Technical note 5. Multidimensional Poverty Index

The Multidimensional Poverty Index (MPI) identifies multiple deprivations at the household level in education, health and standard of living. It uses micro data from household surveys, and—unlike the Inequality-adjusted Human Development Index—all the indicators needed to construct the measure must come from the same survey. More details about the general methodology can be found in Alkire and Santos (2010). More details about changes in the methodology and the treatment of missing responses and non-applicable households are given in Dotter and Klasen (2014) and Kovacevic and Calderon (2014). Programmes (Stata do-files) for computation of the MPI and its components for all countries with appropriate data are available at <http://hdr.undp.org/en/content/mpi-statistical-programmes>.

#### Example: Hungary

	Health		Empowerment		Labour market
	Maternal mortality ratio (deaths per 100,000 live births)	Adolescent birth rate (births per 1,000 women ages 15–19)	Parliamentary representation (percent of seats)	Attainment at secondary and higher education (percent)	Labour market participation rate (percent)
Female	17	18.0	10.1	95.6	46.4
Male	na	na	89.9	97.9	62.5
$\frac{F+M}{2}$	$\sqrt{\left(\frac{10}{17}\right) \cdot \left(\frac{1}{18.0}\right) + 1} = 0.5904$		$\frac{\sqrt{0.101 \cdot 0.956} + \sqrt{0.899 \cdot 0.979}}{2} = 0.6244$		$\frac{0.464 + 0.625}{2} = 0.5445$

na is not applicable.

Using the above formulas, it is straightforward to obtain:

$$G_F: \sqrt[3]{\sqrt{\frac{10}{17} \cdot \frac{1}{18.0}} \cdot \sqrt{0.101 \cdot 0.956} \cdot 0.464} = 0.2965$$

$$G_M: \sqrt[3]{1 \cdot \sqrt{0.899 \cdot 0.979} \cdot 0.625} = 0.8370$$

$$HARM(G_F, G_M): \left[ \frac{1}{2} \left( \frac{1}{0.2965} + \frac{1}{0.8370} \right) \right]^{-1} = 0.4379$$

$$G_{F,M}^-: \sqrt[3]{0.5904 \cdot 0.6244 \cdot 0.5445} = 0.5855$$

$$\text{GII: } 1 - (0.4379/0.5855) = 0.252.$$

### Data sources

- United Nations Children's Fund Multiple Indicator Cluster Surveys.
- ICF Macro Demographic and Health Surveys.
- For several countries, national household surveys with the same or similar content and questionnaires are used: Argentina, 2005 Encuesta Nacional de Nutrición y Salud; Brazil, 2012, 2013 and 2014 Pesquisa Nacional por Amostra de Domicílios; China, 2012 China Family Panel Studies; Ecuador, 2006 and 2014 Encuesta de Condiciones de Vida; Jamaica, 2010 and 2012 Jamaica Survey of Living Conditions; Libya, 2007 Pan Arab Population and Family Health

Survey; Mexico, 2006 and 2012 Encuesta Nacional de Salud y Nutricion; Morocco, 2011 Pan Arab Population and Family Health Survey; South Africa, 2008 and 2012 National Income Dynamics Study; State of Palestine, 2006/2007 Palestinian Family Health Survey; and Syrian Arab Republic, 2009 Pan Arab Population and Family Health Survey.

## Methodology

Each person is assigned a deprivation score according to his or her household's deprivations in each of the 10 component indicators. The maximum deprivation score is 100 percent, with each dimension equally weighted; thus the maximum deprivation score in each dimension is 33.3 percent. The education and health dimensions have two indicators each, so each indicator is worth 33.3/2, or 16.7 percent. The standard of living dimension has six indicators, so each indicator is worth 33.3/6, or 5.6 percent.

The indicator thresholds for households to be considered deprived are as follows:

### Education:

- School attainment: no household member has completed at least six years of schooling.
- School attendance: a school-age child (up to grade 8) is not attending school.<sup>2</sup>

### Health:

- Nutrition: a household member (for whom there is nutrition information) is malnourished, as measured by the body mass index for adults (women ages 15–49 in most of the surveys) and by the height-for-age *z*-score calculated based on World Health Organization standards for children under age 5.
- Child mortality: a child has died in the household within the five years prior to the survey.<sup>3</sup>

### Standard of living:

- Electricity: not having access to electricity.
- Drinking water: not having access to clean drinking water or having access to clean drinking water through a source that is located 30 minutes away or more by walking.
- Sanitation: not having access to improved sanitation facilities or having access only to shared improved sanitation facilities.<sup>4</sup>
- Cooking fuel: using “dirty” cooking fuel (dung, wood or charcoal).
- Having a home with dirt, sand or dung floor.
- Assets: not having at least one asset related to access to information (radio, television or telephone<sup>5</sup>) or having at least one

asset related to information but not having at least one asset related to mobility (bike, motorbike, car, truck, animal cart or motorboat) or at least one asset related to livelihood (refrigerator, arable land<sup>6</sup> or livestock<sup>7</sup>).

To identify the multidimensionally poor, the deprivation scores for each indicator are summed to obtain the household deprivation score. A cutoff of 33.3 percent, which is equivalent to 1/3 of the weighted indicators, is used to distinguish between the poor and nonpoor. If the deprivation score is 33.3 percent or higher, that household (and everyone in it) is multidimensionally poor. Households with a deprivation score of 20 percent or higher but less than 33.3 percent are near multidimensional poverty. Households with a deprivation score of 50 percent or higher are severely multidimensionally poor.

The headcount ratio, *H*, is the proportion of the multidimensionally poor in the population:

$$H = \frac{q}{n}$$

where *q* is the number of people who are multidimensionally poor and *n* is the total population.

The intensity of poverty, *A*, reflects the proportion of the weighted component indicators in which, on average, poor people are deprived. For poor households only (deprivation score *c* of 33.3 percent or higher), the deprivation scores are summed and divided by the total number of poor people:

$$A = \frac{\sum_1^q c_i}{q}$$

where *c<sub>i</sub>* is the deprivation score that the *i*th poor person experiences.

The deprivation score *c<sub>i</sub>* of the *i*th poor person can be expressed as the sum of the weights associated with each indicator *j* (*j* = 1, 2, ..., 10) in which person *i* is deprived, *c<sub>i</sub>* = *c<sub>i1</sub>* + *c<sub>i2</sub>* + ... + *c<sub>i10</sub>*.

The MPI value is the product of two measures: the multidimensional poverty headcount ratio and the intensity of poverty.

$$MPI = H \cdot A$$

The contribution of dimension *k* to multidimensional poverty can be expressed as

$$Contrib_k = \frac{\sum_{j \in k} \sum_1^q c_{ij}}{n} / MPI$$

### Example using hypothetical data

Indicator	Indicator weights	Household			
		1	2	3	4
Household size		4	7	5	4
<b>Education</b>					
No one has completed six years of schooling	$(\frac{1}{3}) \div 2 = 16.7\%$	0	1	0	1
At least one school-age child not enrolled in school	$(\frac{1}{3}) \div 2 = 16.7\%$	0	1	0	0
<b>Health</b>					
At least one member is malnourished	$(\frac{1}{3}) \div 2 = 16.7\%$	0	0	1	0
One or more children have died	$(\frac{1}{3}) \div 2 = 16.7\%$	1	1	0	1
<b>Living conditions</b>					
No electricity	$(\frac{1}{3}) \div 6 = 5.6\%$	0	1	1	1
No access to clean drinking water	$(\frac{1}{3}) \div 6 = 5.6\%$	0	0	1	0
No access to adequate sanitation	$(\frac{1}{3}) \div 6 = 5.6\%$	0	1	1	0
House has dirt floor	$(\frac{1}{3}) \div 6 = 5.6\%$	0	0	0	0
Household uses "dirty" cooking fuel (dung, firewood or charcoal)	$(\frac{1}{3}) \div 6 = 5.6\%$	1	1	1	1
Household has no access to information and has no assets related to mobility or assets related to livelihood.	$(\frac{1}{3}) \div 6 = 5.6\%$	0	1	0	1
<b>Results</b>					
Household deprivation score, <i>c</i> (sum of each deprivation multiplied by its weight)		22.2%	72.2%	38.9%	50.0%
Is the household poor ( <i>c</i> ≥ 33.3 percent)?		No	Yes	Yes	Yes

Note: 1 indicates deprivation in the indicator; 0 indicates nondeprivation.

### Weighted deprivations:

- Household 1:  $(1 \cdot 16.67) + (1 \cdot 5.56) = 22.2$  percent.
- Household 2: 72.2 percent.
- Household 3: 38.9 percent.
- Household 4: 50.0 percent.

Based on this hypothetical population of four households:

**Headcount ratio** ( $H$ ) =

$$\left( \frac{0 + 7 + 5 + 4}{4 + 7 + 5 + 4} \right) = 0.800$$

(80 percent of people live in poor households).

**Intensity of poverty** ( $A$ ) =

$$\frac{(72.2 \cdot 7) + (38.9 \cdot 5) + (50.0 \cdot 4)}{(7 + 5 + 4)} = 56.3 \text{ percent}$$

(the average poor person is deprived in 56.3 percent of the weighted indicators).

$$\text{MPI} = H \cdot A = 0.8 \cdot 0.563 = 0.450.$$

**Contribution of deprivations in:**

Education:

$$\text{contrib}_1 = \frac{16.67 \cdot (7 + 4) + 16.67 \cdot 7}{4 + 7 + 5 + 4} / 45.0 = 33.3 \text{ percent}$$

Health:

$$\text{contrib}_2 = \frac{16.67 \cdot 5 + 16.67 \cdot (7 + 4)}{4 + 7 + 5 + 4} / 45.0 = 29.6 \text{ percent}$$

Living conditions:

$$\text{contrib}_3 = \frac{5.56 \cdot (7 \cdot 4 + 5 \cdot 4 + 4 \cdot 3)}{4 + 7 + 5 + 4} / 45.0 = 37.1 \text{ percent.}$$

Calculating the contribution of each dimension to multidimensional poverty provides information that can be useful for revealing a country's deprivation structure and can help with policy targeting.

## Technical note 6. Life-course Gender Gap Dashboard

The life-course gender gap dashboard focuses on gender gaps and women's empowerment throughout the life-course—childhood and youth, adulthood and older age. It showcases indicators relevant to health, education, labour market and work, empowerment and social protection at different stages of life.

The 14 indicators in the dashboard are:

- **Childhood and youth:** sex ratio at birth, adolescent birth rate, female gross enrolment ratio at different levels of school (pre-primary, primary and secondary) and youth unemployment rate.
- **Adulthood:** maternal mortality ratio; population with at least some secondary education; total unemployment rate;

female share of paid employment in nonagriculture; female share of legislators, senior officials and managers; and share of seats in parliament held by women.

- **Older age:** female life expectancy at age 50 and old-age pension recipients.

### Indicator units and interpretation

Some indicators are expressed in their original units only for women (for example, the female primary school gross

enrolment ratio is presented as the percentage of the female primary school-age population); these indicators emphasize the achievements of girls and women. Others are expressed as a ratio of the value for women to the value for men; these indicators emphasize deviations from expected parity.

The interpretation of the female to male ratios is similar to the interpretation of odds ratios.

Example 1: A ratio of less than 1 for the youth unemployment rate indicates that the unemployment rate for young women (the percentage of the female labour force population ages 15–24 that is not in paid employment or self-employed but is available for work and is actively seeking paid employment or self-employment) is lower than the unemployment rate for young men (the percentage of the male labour force population ages 15–24 that is not in paid employment or self-employed but is available for work and is actively seeking paid employment or self-employment). A ratio of higher than 1 indicates that the unemployment rate for young women is higher than the unemployment rate of young men. The ratio for Canada, 0.88, means that the unemployment rate for young women is about 88 percent of the unemployment rate for young men—or that the odds that a young unemployed person is female is 0.88 times the odds that a young unemployed person is a man. The ratio for Singapore, 1.32, means that the unemployment rate for young women is about 32 percent higher than the rate for young men—or that the odds that a young unemployed person is female is 1.32 times the odds that a young unemployed person is male.

Example 2: A ratio of less than 1 for old-age pension recipients indicates that the percentage of women of the statutory pensionable age receiving an old-age pension is lower than the percentage of men of the statutory pensionable age receiving an old-age pension. Similarly, a ratio of higher than 1 indicates that the percentage of women of statutory pensionable age receiving an old-age pension is higher than the percentage of men of statutory pensionable age receiving an old-age pension. The ratio for Mexico, 0.5, means that the percentage of women of statutory pensionable age receiving an old-age pension is about 50 percent of the percentage of men of statutory

pensionable age receiving an old-age pension—or that the odds that a person of statutory pensionable age receiving an old-age pension is a woman is about 50 percent of the odds that a person of statutory pensionable age receiving an old-age pension is a man. The ratio for the Republic of Moldova, 1.21, means that the odds that a person of statutory pensionable age receiving an old-age pension is a woman is 1.21 times the odds that a person of statutory pensionable age receiving an old-age pension is a man.

## Partial grouping and colour coding

The life-course gender gap dashboard allows partial grouping of countries by each indicator rather than complete grouping by a composite measure such as the Human Development Index that combines the listed indicators after making them commensurable. Partial grouping by indicator does not require assumptions about normalization, weighting or the functional form of the composite index. The dashboard does not use predefined thresholds or target values. For each indicator countries are divided into three groups of approximately equal sizes (terciles); the groups are referred to as the top third, the middle third and the bottom third. The only exception is the sex ratio at birth indicator, for which countries are divided into two groups: one group for countries with a sex ratio at birth in the natural range (between 1.04 and 1.07) and one group for countries with a gender-biased sex ratio at birth (that is, a ratio outside the natural range).

Countries in each group are shaded the same colour, with a darker shade used for the top third, a medium shade used for the middle third and a light shade used for the bottom third. For indicators that are expressed as a female to male ratio, countries with values in the vicinity of 1 are in the top third, and large gaps in favour of men or women are treated the same way. For some highly skewed distributions (such as female primary gross enrolment ratio and ratio of female to male unemployment rate) the groups differ greatly in size.

### Observed ranges of values and number of countries in each tercile group, by indicator, life-course gender gap dashboard

Indicator	Top group		Middle group		Bottom group		Countries with missing values
	Range	Number of countries	Range	Number of countries	Range	Number of countries	
Sex ratio at birth (male to female births)	1.04–1.07	135	—	—	<1.04, >1.07	50	10
Adolescent birth rate (births per 1,000 women ages 15–19)	<20	61	20–60	63	>60	61	10
Gross enrolment ratio, preprimary, female (% of preschool-age female population)	≥80	64	40–80	51	<40	56	24
Gross enrolment ratio, primary, female (% of primary school-age female population)	≥99	127	95–99	19	<95	34	15
Gross enrolment ratio, secondary, female (% of secondary school-age female population)	≥95	73	75–95	44	<75	52	26
Youth unemployment rate (female to male ratio)	0.94–1.06	49	0.85–0.94, 1.06–1.15	64	<0.85, >1.15	65	17
Maternal mortality ratio (deaths per 100,000 live births)	<25	64	25–140	57	>140	61	13
Population with at least some secondary education (female to male ratio)	0.96–1.04	57	0.85–0.96, 1.04–1.15	52	<0.85, >1.15	55	31
Total unemployment rate (female to male ratio)	0.90–1.10	33	0.80–0.90, 1.10–1.20	37	<0.80, >1.20	108	17
Share of paid employment in nonagriculture, female (% of total paid employment in nonagriculture)	≥47	44	40–47	31	<40	33	87
Female legislators, senior officials and managers (% of total)	≥35	42	25–35	34	<25	40	79
Share of seats in parliament (% held by women)	≥25	62	15–25	63	<15	67	3
Life expectancy at age 50, female (years)	≥31	63	27–31	55	<27	67	10
Old-age pension recipients (female to male ratio)	0.95–1.05	32	0.80–0.95, 1.05–1.20	13	<0.80, >1.20	33	117

## Technical note 7. Sustainable Development Dashboard

The sustainable development dashboard focuses on sustainability in the environmental, economic and social realms.

The 15 indicators in the dashboard are:

- **Environmental sustainability:** renewable energy consumption, carbon dioxide emissions per capita, average annual change in carbon dioxide emissions, forest area as percentage of total land area, change in forest area, and fresh water withdrawals.
- **Economic sustainability:** natural resource depletion, adjusted net savings, external debt stock, research and development expenditure and concentration of exports index.
- **Social sustainability:** income quintile ratio, Gender Inequality Index, population in multidimensional poverty and old-age dependency ratio.

### Indicator units

Except for average annual change in carbon dioxide emissions and change in forest area, indicators of environmental and economic sustainability are expressed in their original units. Except for old-age dependency ratio, indicators of social sustainability are expressed as average annual change over a specified period.

### Partial grouping and colour coding

The sustainable development dashboard allows partial grouping of countries by each indicator rather than complete grouping by a composite measure such as the Human Development Index that combines the listed indicators after making them commensurable. Partial grouping by indicator does not require

assumptions about normalization, weighting or the functional form of the composite index. The dashboard does not use pre-defined thresholds or target values. For each indicator countries are divided into three groups of approximately equal sizes (terciles); the groups are referred to as the top third, the middle third and the bottom third. The only exception is the indicator on forest area as percentage of total land area, which is not used

for comparison (because forest area depends on many geological, geographic and historical factors beyond the undertakings of the current generation) but is meant to provide context for the indicator on change in forest area, which provides a measure of the importance of forests in a country or a region and aids in monitoring planned and unplanned deforestation as well as restoration and rehabilitation of forests.

*Observed ranges of values and number of countries in each tercile group, by indicator, sustainable development dashboard*

Indicator	Top group		Middle group		Bottom group		Countries with missing values
	Range	Number of countries	Range	Number of countries	Range	Number of countries	
Renewable energy consumption (% of total final energy consumption)	>40.0	60	12.0 to 40.0	59	<12.0	59	17
Carbon dioxide emissions, per capita (tonnes)	≤ 1.2	65	1.2 to 5.0	65	>5.0	62	3
Carbon dioxide emissions, average annual change (%)	≤ 0.4	57	0.4 to 2.3	45	>2.3	57	36
Forest area, change (%)	≥4.5	61	-5.0 to 4.5	62	<-5.0	61	11
Fresh water withdrawals (% of total renewable water resources)	≤ 2.6	44	2.6 to 15.0	50	>15.0	38	63
Natural resource depletion (% of GNI)	≤ 1.0	59	1.0 to 6.2	56	>6.2	55	25
Adjusted net savings (% of GNI)	≥ 13.0	51	3.5 to 13.0	49	<3.5	51	44
External debt stock (% of GNI)	≤25.0	36	25.0 to 50.0	43	>50.0	39	77
Research and development expenditure (% of GDP)	≥0.8	40	0.3 to 0.8	40	<0.3	42	73
Concentration index (exports)	≤ 0.200	66	0.200 to 0.400	59	>0.400	65	5
Income quintile ratio, average annual change (%)	≤-1.5	34	-1.5 to 0.15	30	>0.15	32	99
Gender Inequality Index, average annual change (%)	≤-2.0	53	-2.0 to -1.0	41	>-1.0	44	57
Population in multidimensional poverty, average annual change (%)	≤-6.0	19	-6.0 to -1.3	23	>-1.3	21	132
Old-age (ages 65 and older) dependency ratio (per 100 people ages 15-64)	≤9.0	61	9.0 to 22.0	63	>22.0	61	10

## Notes

1. The inequality aversion parameter affects the degree to which lower achievements are emphasized and higher achievements are de-emphasized.
2. Up to one year late enrollment to primary school is allowed for to prevent counting a mismatch between the birthday and the beginning of the school year as a deprivation.
3. Some surveys do not collect information about time when the death of a child occurred; in such cases any child death reported by a mother age 35 or younger is counted.
4. Drinking water and improved sanitation are as defined in the Millennium Development Goals.
5. Including both landline and mobile telephones.
6. Any size of land usable for agriculture.
7. A horse, a head of cattle, two goats, two sheep or 10 chickens.

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