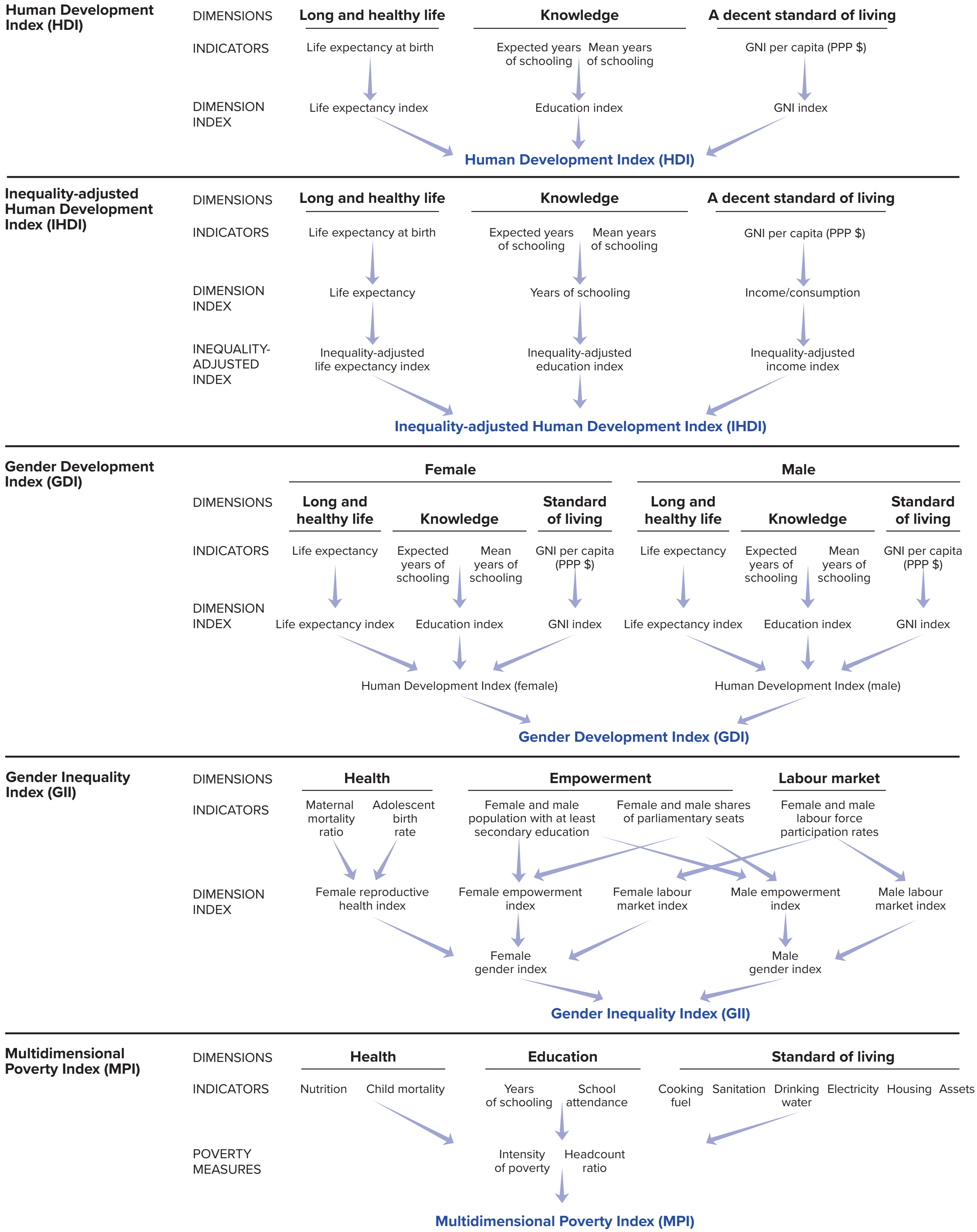
Technical notes

Calculating the human development indices—graphical presentation



Note: See figure 1 on page 14 for a graphical presentation of the Planetary pressures–adjusted Human Development Index (PHDI).

Technical note 1. Human Development Index

The Human Development Index (HDI) is a summary measure of achievements in three key dimensions of human development: a long and healthy life, access to knowledge and a decent standard of living. The HDI is the geometric mean of normalized indices for each of the three dimensions. This technical note describes the data sources and the steps to calculating HDI values.

Data sources

- Life expectancy at birth: UNDESA (2022a).
- Expected years of schooling: CEDLAS and World Bank (2022), ICF Macro Demographic and Health Surveys (various years), UNESCO Institute for Statistics (2022) and United Nations Children's Fund (UNICEF) Multiple Indicator Cluster Surveys (various years).
- Mean years of schooling: Barro and Lee (2018), ICF Macro Demographic and Health Surveys (various years), OECD (2022), UNESCO Institute for Statistics (2022) and UNICEF Multiple Indicator Cluster Surveys (various years).
- GNI per capita: IMF (2022), UNDESA (2022b), United Nations Statistics Division (2022) and World Bank (2022).

Steps to calculate Human Development Index values

There are two steps to calculating HDI values.

Step 1. Creating the dimension indices

Minimum and maximum values (goalposts) are set in order to transform the indicators expressed in different units into indices between 0 and 1. These goalposts act as "the natural zeros" and "aspirational targets," respectively, from which component indicators are standardized (see equation 1 below). They are set at the following values:

Dimension	Indicator	Minimum	Maximum
Health	Life expectancy at birth (years)	20	85
Education	Expected years of schooling (years)	0	18
	Mean years of schooling (years)	0	15
Standard of living	GNI per capita (2017 PPP$)	100	75,000

The justification for placing the natural zero for life expectancy at birth at 20 years is based on historical evidence that no country in the 20th century had a life expectancy at birth of less than 20 years (Maddison 2010; Oeppen and Vaupel 2002; Riley 2005). Maximum life expectancy at birth is set at 85, a realistic aspirational target for many countries over the last 30 years. Due to constantly improving living conditions and medical advances, life expectancy at birth in several economies has already exceeded or come very close to 85 years: 85.9 years in Monaco, 85.5 years in Hong Kong, China (Special Administrative Region) and 84.8 years in Japan.

Societies can subsist without formal education, justifying the education minimum of 0 years. The maximum for expected years of schooling, 18, is equivalent to achieving a master's degree in most countries. The maximum for mean years of schooling, 15, is the projected maximum of this indicator for 2025.

The low minimum value for gross national income (GNI) per capita, $100, is justified by the considerable amount of unmeasured subsistence and nonmarket production in economies close to the minimum, which is not captured in the official data. The maximum is set at $75,000 per capita. Kahneman and Deaton (2010) have shown that there is virtually no gain in human development and wellbeing from annual income above $75,000 per capita. Currently, only five countries (Liechtenstein, Singapore, Qatar, Luxembourg and Ireland) exceed the $75,000 income per capita ceiling.

Having defined the minimum and maximum values, the dimension indices are calculated as:

$$\text{Dimension index} = \frac{\text{actual value} - \text{minimum value}}{\text{maximum value} - \text{minimum value}}. \quad (1)$$

For the education dimension, equation 1 is first applied to each of the two indicators, and then the arithmetic mean of the two resulting indices is taken. Using the arithmetic mean of the two education indices allows perfect substitutability between expected years of schooling and mean years of schooling, which seems to be right given that many developing countries have low school attainment among adults but are very eager to achieve universal primary and secondary school enrolment among school-age children.

Because each dimension index is a proxy for capabilities in the corresponding dimension, the transformation function from income to capabilities is likely to be concave (Anand and Sen 2000)—that is, each additional dollar of income has a smaller effect on expanding capabilities. Thus for income the natural logarithm of the actual, minimum and maximum values is used.

Step 2. Aggregating the dimensional indices

The HDI is the geometric mean of the three dimensional indices:

$$HDI = (I_{Health} \cdot I_{Education} \cdot I_{Income})^{1/3}$$

Example: Guyana

Indicator	Value
Life expectancy at birth (years)	65.7
Expected years of schooling (years)	12.5
Mean years of schooling (years)	8.6
Gross national income per capita (2017 PPP $)	22,465

Note: Values are rounded.

$$\text{Health index} = \frac{65.7 - 20}{85 - 20} = 0.703$$

$$\text{Expected years of schooling index} = \frac{12.5 - 0}{18 - 0} = 0.694$$

$$\text{Mean years of schooling index} = \frac{8.6 - 0}{15 - 0} = 0.573$$

$$\text{Education index} = \frac{0.694 + 0.573}{2} = 0.634$$

$$\text{Income index} = \frac{\ln(22{,}465) - \ln(100)}{\ln(75{,}000) - \ln(100)} = 0.818$$

Human Development Index =
$(0.703 \cdot 0.634 \cdot 0.818)^{1/3} = 0.714$

Methodology used to express income

The World Bank's 2022 World Development Indicators database contains estimates of GNI per capita in constant 2017 purchasing power parity (PPP) terms for many countries. For countries missing this indicator (entirely or partly), the Human Development Report Office (HDRO) calculates it by converting GNI per capita in local currency from current to constant terms using two steps. First, the value of GNI per capita in current terms is converted into PPP terms for the base year (2017). Second, a time series of GNI per capita in 2017 PPP constant terms is constructed by applying the real growth rates to the GNI per capita in PPP terms for the base year. The real growth rate is implied by the ratio of the nominal growth of GNI per capita in current local currency terms to the GDP deflator.

For several countries without a value of GNI per capita in constant 2017 PPP terms for 2021 reported in the World Development Indicators database, real growth rates of GDP per capita available in the World Development Indicators database or in the International Monetary Fund's Economic Outlook database are applied to the most recent GNI values in constant PPP terms.

Official PPP conversion rates are produced by the International Comparison Program, whose surveys periodically collect thousands of prices of matched goods and services in many countries. The last round of this exercise refers to 2017 and covered 176 economies.

Human development categories

The 2014 Human development Report introduced a system of fixed cutoff points for the four categories of human development achievements. The cutoff points *(COP)* are the HDI values calculated using the quartiles *(q)* from the distributions of the component indicators averaged over 2004–2013:

$$COP_q = HDI\,(LE_q, EYS_q, MYS_q, GNIpc_q),\ q = 1,2,3.$$

For example, LE_1, LE_2 and LE_3 denote three quartiles of the distribution of life expectancy at birth across countries.

This Report keeps the same cutoff points on the HDI for grouping countries that were introduced in the 2014 Report:

Very high human development	0.800 and above
High human development	0.700–0.799
Medium human development	0.550–0.699

Low human development	Below 0.550

Human Development Index aggregates

Aggregate HDI values for country groups (by human development category, region and the like) are calculated by applying the HDI formula to the weighted group averages of component indicators. Life expectancy at birth and GNI per capita are weighted by total population, expected years of schooling is weighted by population ages 5–24 and mean years of schooling is weighted by population ages 25 and older.

Technical note 2. Inequality-adjusted Human Development Index

The Inequality-adjusted Human Development Index (IHDI) adjusts the Human Development Index (HDI) for inequality in the distribution of each dimension across the population. It is based on a distribution-sensitive class of composite indices proposed by Foster, Lopez-Calva and Szekely (2005), which draws on the Atkinson (1970) family of inequality measures. It is computed as a geometric mean of inequality-adjusted dimensional indices.

The IHDI accounts for inequalities in HDI dimensions by "discounting" each dimension's average value according to its level of inequality. The IHDI value equals the HDI value when there is no inequality across people but falls below the HDI value as inequality rises. In this sense the IHDI measures the level of human development when inequality is accounted for.

Data sources

Since the HDI relies on country-level aggregates such as national accounts for income, the IHDI must draw on additional sources of data to obtain insights into the distribution. The distributions are observed over different units—life expectancy is distributed across a hypothetical cohort, while years of schooling and income are distributed across individuals.

Inequality in the distribution of HDI dimensions is estimated for:

- Life expectancy, using data from complete life tables provided by UNDESA (2022a). Mortality rates and other information are available for each age (0, 1, 2, 3, ... 100+). This is a major update: in previous Reports the inequality in the distribution of life expectancy was computed based on abridged life tables, which presented information on mortality patterns over age intervals (0–1, 1–5, 5–10, ... 100+).
- Mean years of schooling, using household surveys data harmonized in international databases, including the Luxembourg Income Study, Eurostat's European Union Survey of Income and Living Conditions, the World Bank's International Income Distribution Database, ICF Macro's Demographic and Health Surveys, United Nations Children's Fund's Multiple Indicators Cluster Surveys, the Center for Distributive, Labour and Social Studies and the World Bank's Socio-Economic Database for Latin America and the Caribbean, the United Nations Educational, Scientific and Cultural Organization Institute for Statistics' Educational Attainment Table and the United Nations University's World Income Inequality Database.
- Disposable household income or consumption per capita using the above listed databases and household surveys—and for some countries, income imputed based on an asset index matching methodology using household survey asset indices (Harttgen and Vollmer 2013). The asset index is provided in microdata from ICF Macro Demographic and Health Surveys and United Nations Children's Fund Multiple Indicator Cluster Surveys.

A full account of data sources used for estimating inequality for the 2021 IHDI is available at https://hdr.undp.org/inequality-adjusted-human-development-index#/indicies/IHDI.

Steps to calculate Inequality-adjusted Human Development Index values

There are three steps to calculating IHDI values.

Step 1. Estimating inequality in the three dimensions of the Human Development Index

The IHDI draws on the Atkinson (1970) family of inequality measures and sets the aversion parameter ε equal to 1.[1] In this case the inequality measure is $A = 1- g/\mu$, where g is the geometric mean and μ is the arithmetic mean of the distribution. This can be written as:

$$A_x = 1 - \frac{\sqrt[n]{X_1 \ldots X_n}}{\bar{X}} \qquad (1)$$

where $\{X_1, \ldots, X_n\}$ denotes the underlying distribution in the dimension of interest. A_x is obtained for each variable (life expectancy, mean years of schooling and disposable household income or consumption per capita).

The geometric mean in equation 1 does not allow zero values. For mean years of schooling one year is added to all valid observations to compute the inequality. For income per capita, when the Atkinson measure is estimated by HDRO, negative and zero values and values in the bottom 0.5 percentile are replaced with the minimum value of the second bottom 0.5 percentile of the distribution of positive incomes. The top 0.5 percentile of the distribution is truncated to reduce the impact of measurement errors when recording extremely high incomes. Sensitivity analysis of the IHDI is given in Kovacevic (2010). Atkinson measures based on income or consumption from other sources may follow a different trimming approach.

Step 2. Adjusting the dimension indices for inequality

The inequality-adjusted dimension indices are obtained from the HDI dimension indices, I_x, by multiplying them by $(1 - A_x)$, where A_x, defined by equation 1, is the corresponding Atkinson measure:

$$I^*_x = (1 - A_x) \cdot I_x .$$

The inequality-adjusted income index, I^*_{income}, is based on the index of logged income values, I_{income^*}, and inequality in income distribution computed using income in levels. This enables the IHDI to account for the full effect of income inequality.

1 The inequality aversion parameter affects the degree to which lower achievements are emphasized and higher achievements are de-emphasized.

Step 3. Combining the dimension indices

The IHDI is the geometric mean of the three dimension indices adjusted for inequality:

$$IHDI = (I^*_{Health} \cdot I^*_{Education} \cdot I^*_{Income})^{1/3} = [(1 - A_{Health}) \cdot (1 - A_{Education}) \cdot (1 - A_{Income})]^{1/3} \cdot HDI.$$

The loss in HDI value due to inequality is:

$$Loss = 1 - [(1 - A_{Health}) \cdot (1 - A_{Education}) \cdot (1 - A_{Income})]^{1/3}.$$

Difference in Inequality-adjusted Human Development Index rank and Human Development Index rank

Difference in ranks on the IHDI and the HDI is calculated only for countries for which both an IHDI value and an HDI value are calculated. Due to data limitations, IHDI values are calculated for a smaller set of countries. To calculate the IHDI rank difference from the HDI rank, HDI ranks are recalculated for the set of countries for which an IHDI value is calculated.

Coefficient of human inequality

An unweighted average of inequalities in health, education and income is denoted as the coefficient of human inequality. It averages these inequalities using the arithmetic mean:

$$Coefficient\ of\ human\ inequality = \frac{A_{Health} + A_{Education} + A_{Income}}{3}.$$

When all inequalities in dimensions are of a similar magnitude, the coefficient of human inequality and the loss in HDI value differ negligibly. When inequalities differ in magnitude, the loss in HDI value tends to be higher than the coefficient of human inequality.

Notes on methodology and caveats

The IHDI is based on the Atkinson index, which satisfies subgroup consistency. This property ensures that improvements (deteriorations) in the distribution of human development within only a certain group of the society imply improvements (deteriorations) in the distribution across the entire society.

The main disadvantage is that the IHDI is not association sensitive, so it does not capture overlapping inequalities. To make the measure association sensitive, all the data for each individual must be available from a single survey source, which is not currently possible for a large number of countries.

Example: Kazakhstan

Indicator	Indicator	Dimension index	Inequality measure (A)[a]	Inequality-adjusted index (I^*)
Life expectancy (years)	69.4	0.7594	0.073	$(1-0.073) \cdot 0.7594 = 0.7042$
Expected years of schooling (years)	15.8	0.8758	—	—
Mean years of schooling (years)	12.3	0.8232	0.032	—
Education index		0.8495	0.032	$(1-0.032) \cdot 0.8495 = 0.8226$
Gross national income per capita (2017 PPP $)	23,943	0.8275	0.103	$(1-0.103) \cdot 0.8275 = 0.742$

Human Development Index	Inequality-adjusted Human Development Index
$(0.7594 \cdot 0.8495 \cdot 0.8275)^{1/3} = 0.811$	$(0.7042 \cdot 0.8226 \cdot 0.742)^{1/3} = 0.755$
Loss due to inequality (%):	**Coefficient of human inequality (%)**
$100 \cdot \left(1 - \frac{0.755}{0.811}\right) = 6.9$	$\frac{100 \cdot (0.073 + 0.032 + 0.103)}{3} = 6.9$

Note: Values are rounded.
a. Inequalities are estimated from micro data.

Technical note 3. Gender Development Index

The Gender Development Index (GDI) measures gender inequalities in achievement in three basic dimensions of human development: health, measured by female and male life expectancy at birth; education, measured by female and male expected years of schooling for children and female and male mean years of schooling for adults ages 25 years and older; and command over economic resources, measured by female and male estimated earned income.

Data sources

- Life expectancy at birth: UNDESA (2022a).
- Expected years of schooling: CEDLAS and World Bank (2022), ICF Macro Demographic and Health Surveys (various years), UNESCO Institute for Statistics (2022) and United Nations Children's Fund (UNICEF) Multiple Indicator Cluster Surveys (various years).
- Mean years of schooling for adults ages 25 and older: Barro and Lee (2018), ICF Macro Demographic and Health Surveys (various years), OECD (2022), UNESCO Institute for Statistics (2022) and UNICEF Multiple Indicator Cluster Surveys (various years).
- Estimated earned income: Human Development Report Office estimates based on female and male shares of the economically active population, the ratio of the female to male wage in all sectors and gross national income in 2017 purchasing power parity (PPP) terms, and female and male shares of population from ILO (2022), IMF (2022), UNDESA (2022a), United Nations Statistics Division (2022) and World Bank (2022).

Steps to calculate Gender Development Index values

There are four steps to calculating GDI values.

Step 1. Estimating female and male earned incomes

To calculate estimated earned incomes, the share of the wage bill is calculated for each gender. The female share of the wage bill (S_f) is calculated as follows:

$$S_f = \frac{W_f/W_m \cdot EA_f}{W_f/W_m \cdot EA_f + EA_m}$$

where Wf/Wm is the ratio of female to male wage, EA_f is the female share of the economically active population and EA_m is the male share.

The male share of the wage bill is calculated as:

$$S_m = 1 - S_f.$$

Estimated female earned income per capita ($GNIpc_f$) is obtained from GNI per capita ($GNIpc$), first by multiplying it by the female share of the wage bill, S_f, and then rescaling it by the female share of the population, $P_f = N_f/N$:

$$GNIpc_f = GNIpc \cdot S_f/P_f.$$

Estimated male earned income per capita is obtained in the same way:

$$GNIpc_m = GNIpc \cdot S_m/P_m$$

where $P_m = 1 - P_f$ is the male share of population.

Step 2. Normalizing the indicators

To construct the female and male HDI values, first the indicators, which are in different units, are transformed into indices and then dimension indices for each sex are aggregated by taking the geometric mean.

The indicators are transformed into indices on a scale of 0 to 1 using the same goalposts that are used for the HDI, except life expectancy at birth, which is adjusted for the average five-year biological advantage that women have over men.

Goalposts for the Gender Development Index in this Report

Indicator	Minimum	Maximum
Life expectancy at birth (years)		
Female	22.5	87.5
Male	17.5	82.5
Expected years of schooling (years)	0	18
Mean years of schooling (years)	0	15
Estimated earned income (2017 PPP $)	100	75,000

Note: For the rationale on choice of minimum and maximum values, see *Technical note 1.*

Having defined the minimum and maximum values, the subindices are calculated as follows:

$$\text{Dimension index} = \frac{\text{actual value} - \text{minimum value}}{\text{maximum value} - \text{minimum value}}.$$

For education the dimension index is first obtained for each of the two subcomponents, and then the unweighted arithmetic mean of the two resulting indices is taken.

Step 3. Calculating the female and male Human Development Index values

The female and male HDI values are the geometric means of the three dimensional indices for each gender:

$$HDI_f = (I_{Health_f} \cdot I_{Education_f} \cdot I_{Income_f})^{1/3}$$

$$HDI_m = (I_{Health_m} \cdot I_{Education_m} \cdot I_{Income_m})^{1/3}$$

Step 4. Comparing female and male Human Development Index values

The GDI is simply the ratio of female HDI value to male HDI value:

$$GDI = \frac{HDI_f}{HDI_m}.$$

Example: Mauritania

Indicator	Female value	Male value
Life expectancy at birth (years)	66.1	62.7
Expected years of schooling (years)	9.6	9.2
Mean years of schooling (years)	4.6	5.3
Wage ratio (female/male)	0.800	
Gross national income per capita (2017 PPP $)	5,075.306	
Share of economically active population	0.307	0.693
Share of population	0.51016	0.48984

Female wage bill:
$S_f = (0.800 \cdot 0.307) / [(0.800 \cdot 0.307) + 0.693] = 0.2617$

Estimated female earned income per capita:
$GNIpc_f = 5{,}075.306 \cdot 0.2617 / 0.51016 = 2{,}604$

Male wage bill:
$S_m = 1 - 0.2617 = 0.7383$

Estimated male earned income per capita:
$GNIpc_m$ = 5,075.306 · 0.7383 / 0.48984 = 7,650

Female health index = (66.1 – 22.5) / (87.5 – 22.5) = 0.6708

Male health index = (62.7 – 17.5) / (82.5 – 17.5) = 0.6954

Female education index = [(9.6 / 18) + (4.6 / 15)] / 2 = 0.42

Male education index = [(9.2 / 18) + (5.3 / 15)] / 2 = 0.4322

Estimated female earned income index:
[ln(2,604) – ln(100)] / [ln(75,000) – ln(100)] = 0.4924

Estimated male earned income index:
[ln(7,650) – ln(100)] / [(ln(75,000) – ln(100)] = 0.6552

Female HDI = $(0.6708 \cdot 0.42 \cdot 0.4924)^{1/3}$ = 0.518

Male HDI = $(0.6954 \cdot 0.4322 \cdot 0.6552)^{1/3}$ = 0.582

GDI = 0.518 / 0.582 = 0.890

Note: Values are rounded.

Gender Development Index groups

The GDI groups are based on the absolute deviation of GDI from gender parity, 100 · |*GDI* – 1|. Countries with absolute deviation from gender parity of 2.5 percent or less are considered countries with high equality in HDI achievements between women and men and are classified as group 1. Countries with absolute deviation from gender parity of 2.5–5 percent are considered countries with medium-high equality in HDI achievements between women and men and are classified as group 2. Countries with absolute deviation from gender parity of 5–7.5 percent are considered countries with medium equality in HDI achievements between women and men and are classified as group 3. Countries with absolute deviation from gender parity of 7.5–10 percent are considered countries with medium-low equality in HDI achievements between women and men and are classified as group 4. Countries with absolute deviation from gender parity of more than 10 percent are considered countries with low equality in HDI achievements between women and men and are classified as group 5.

Technical note 4. Gender Inequality Index

The Gender Inequality Index (GII) reflects gender-based disadvantage in three dimensions—reproductive health, empowerment and the labour market—for as many countries as data of reasonable quality allow. It shows the loss in potential human development due to inequality between female and male achievements in these dimensions. It ranges from 0, where women and men fare equally, to 1, where one gender fares as poorly as possible in all measured dimensions.

GII values are computed using the association-sensitive inequality measure suggested by Seth (2009), which implies that the index is based on the general mean of general means of different orders—the first aggregation is by a geometric mean across dimensions; these means, calculated separately for women and men, are then aggregated using a harmonic mean across genders.

Data sources

- Maternal mortality ratio (*MMR*): WHO, UNICEF, UNFPA, World Bank Group and United Nations Population Division (2019).
- Adolescent birth rate (*ABR*): UNDESA (2022a).
- Share of parliamentary seats held by each sex (*PR*): IPU (2022).
- Population with at least some secondary education (*SE*): Barro and Lee (2018), ICF Macro Demographic and Health Surveys (various years), OECD (2022), UNESCO Institute for Statistics (2022) and United Nations Children's Fund Multiple Indicator Cluster Surveys (various years).
- Labour force participation rate (*LFPR*): ILO (2022).

Steps to calculate Gender Inequality Index values

There are five steps to calculating GII values.

Step 1. Treating zeros and extreme values

Because a geometric mean cannot be computed from zero values, a minimum value of 0.1 percent is set for all component indicators. Further, as higher maternal mortality suggests poorer maternal health, for the maternal mortality ratio the maximum value is truncated at 1,000 deaths per 100,000 births and the minimum value at 10. The rationale is that countries where maternal mortality ratios exceed 1,000 do not differ in their inability to create conditions and support for maternal health and that countries with 10 or fewer deaths per 100,000 births are performing at essentially the same level and that small differences are random. Sensitivity analysis of the GII is given in Gaye and others (2010).

Step 2. Aggregating across dimensions within each gender group, using geometric means

Aggregating across dimensions for each gender group by the geometric mean makes the GII association sensitive (see Seth 2009).

For women and girls the aggregation formula is:

$$G_F = \sqrt[3]{\left(\frac{10}{MMR}\cdot\frac{1}{ABR}\right)^{1/2}\cdot(PR_F\cdot SE_F)^{1/2}\cdot LFPR_F}\,, \qquad (1)$$

and for men and boys the formula is

$$G_M = \sqrt[3]{1\cdot(PR_M\cdot SE_M)^{1/2}\cdot LFPR_M}\,.$$

The rescaling by 0.1 of the maternal mortality ratio in equation 1 is needed to account for the truncation of the maternal mortality ratio at 10.

Step 3. Aggregating across gender groups, using a harmonic mean

The female and male indices are aggregated by the harmonic mean to create the equally distributed gender index

$$HARM\,(G_F, G_M) = \left[\frac{(G_F)^{-1} + (G_M)^{-1}}{2}\right]^{-1}.$$

Using the harmonic mean of within-group geometric means captures the inequality between women and men and adjusts for association between dimensions—that is, it accounts for the overlapping inequalities in dimensions.

Step 4. Calculating the geometric mean of the arithmetic means for each indicator

The reference standard for computing inequality is obtained by aggregating female and male indices using equal weights (thus treating the genders equally) and then aggregating the indices across dimensions:

$$G_{\bar{F},\bar{M}} = \sqrt[3]{\overline{Health}\cdot\overline{Empowerment}\cdot\overline{LFPR}}$$

where $\overline{Health} = \left(\sqrt{\frac{10}{MMR}\cdot\frac{1}{ABR}}+1\right)/2,$

$$\overline{Empowerment} = \left(\sqrt{PR_F\cdot SE_F} + \sqrt{PR_M\cdot SE_M}\right)/2 \text{ and}$$

$$\overline{LFPR} = \frac{LFPR_F + LFPR_M}{2}\,.$$

$\overline{Health}$ should not be interpreted as an average of corresponding female and male indices but rather as half the distance from the norms established for the reproductive health indicators—fewer maternal deaths and fewer adolescent pregnancies.

Step 5. Comparing indices

Comparing the equally distributed gender index to the reference standard yields the GII,

$$1 - \frac{HARM\,(G_F,\,G_M)}{G_{\bar{F},\bar{M}}}\,.$$

Example: Afghanistan

	Health		Empowerment		Labour market
	Maternal mortality ratio (deaths per 100,000 live births)	Adolescent birth rate (births per 1,000 women ages 15–19)	Share of seats in parliament (% held)	Population with at least some secondary education (%)	Labour force participation rate (%)
Female	638	82.6	27.2	6.4	14.8
Male	na	na	72.8	14.9	66.5
$\frac{F+M}{2}$	$\frac{\sqrt{\left(\frac{10}{638}\right)\cdot\left(\frac{1}{82.6}\right)}+1}{2} = 0.5069$		$\frac{\sqrt{0.272\cdot 0.064}+\sqrt{0.728\cdot 0.149}}{2} = 0.2306$		$\frac{0.148+0.665}{2} = 0.4065$

na is not applicable.

Using the above formulas, it is straightforward to obtain:

$$G_F: \sqrt[3]{\sqrt{\frac{10}{638}\cdot\frac{1}{82.6}}\cdot\sqrt{0.272\cdot 0.064}\cdot 0.148} = 0.0646$$

$$G_M: \sqrt[3]{1\cdot\sqrt{0.728\cdot 0.149}\cdot 0.665} = 0.6028$$

$$HARM\ (G_F, G_M): \left[\frac{1}{2}\left(\frac{1}{0.0646}+\frac{1}{0.6028}\right)\right]^{-1} = 0.1167$$

$$G_{\bar{F},\bar{M}}: \sqrt[3]{0.5069\cdot 0.2306\cdot 0.4065} = 0.3622$$

GII: 1 – (0.1167/0.3622) = 0.678.

Technical note 5. Multidimensional Poverty Index

The global Multidimensional Poverty Index (MPI) identifies multiple deprivations at the household level in health, education and standard of living. It uses micro data from household surveys, and—unlike the Inequality-adjusted Human Development Index—all the indicators needed to construct the measure must come from the same survey. More details about the general methodology can be found in Alkire and Jahan (2018). Programmes (Stata do-files) for computing the MPI and its components for a large selection of countries with appropriate data are available at http://hdr.undp.org/en/content/mpi-statistical-programmes. This year the Human Development Report Office is releasing for the first time programmes for computing the MPI in R format for a selection of countries; it plans to increase the programmes in R in the future.

Data sources

- ICF Macro Demographic and Health Surveys.
- United Nations Children's Fund Multiple Indicator Cluster Surveys.
- For several countries, national household surveys with the same or similar content and questionnaires: Botswana, 2015–2016 Multi-Topic Household Survey; Brazil, 2015 Pesquisa Nacional por Amostra de Domicílios; China, 2014 China Family Panel Studies; Cuba, 2017 Encuesta Nacional de Ocupacion; Ecuador, 2013–2014 Encuesta de Condiciones de Vida; Jamaica, 2014 Jamaica Survey of Living Conditions; Libya, 2014 Pan Arab Population and Family Health Survey; Mexico, 2016 Encuesta Nacional de Salud y Nutricion; Morocco, 2011 Pan Arab Population and Family Health Survey; Peru, 2018 Encuesta Demográfica y de Salud Familiar; Seychelles, 2019 Quarterly Labor Force Survey; Sri Lanka, 2016 Demographic and Health Survey; and Syrian Arab Republic, 2009 Pan Arab Population and Family Health Survey.

Methodology

The 2020 global MPI has the same functional form and indicators as in 2018. It continues to use 10 indicators in three dimensions—health, education and standard of living—following the same dimensions and weights as the Human Development Index.

Each person is assigned a deprivation score according to his or her household's deprivations in each of the 10 indicators. The maximum deprivation score is 100 percent, with each dimension equally weighted; thus, the maximum deprivation score in each dimension is 33.3 percent or, more accurately, 1/3. The health and education dimensions have two indicators each, so each indicator is weighted as 1/6. The standard of living dimension has six indicators, so each indicator is weighted as 1/18.

To identify multidimensionally poor people, the deprivation scores for each indicator are summed to obtain the household deprivation score. A cutoff of

1/3 is used to distinguish between poor and nonpoor people. If the deprivation score is 1/3 or higher, that household (and everyone in it) is considered multidimensionally poor. People with a deprivation score of 1/5 or higher but less than 1/3 are considered to be vulnerable to multidimensional poverty. People with a deprivation score of 1/2 or higher are considered to be in severe multidimensional poverty.

The headcount ratio, H, is the proportion of multidimensionally poor people in the population:

$$H = \frac{q}{n}$$

where q is the number of people who are multidimensionally poor and n is the total population.

The intensity of poverty, A, reflects the average proportion of the weighted component indicators in which multidimensionally poor people are deprived. For multidimensionally poor people only (those with a deprivation score s greater than or equal to 33.3 percent), the deprivation scores are summed and divided by the total number of multidimensionally poor people:

$$A = \frac{\sum_1^q s_i}{q}$$

where s_i is the deprivation score that the ith multidimensionally poor person experiences.

The deprivation score s_i of the ith multidimensionally poor person can be expressed as the sum of the weights associated with each indicator j (j = 1, 2, ..., 10) in which person i is deprived, $s_i = c_{i1} + c_{i2} + ... + c_{i10}$.

Dimension	Indicator	Deprived if...	Weight
Health	Nutrition	Any adult under age 70 years or any child for whom nutritional information is available is undernourished. Adults ages 19–70 years (229–840 months) are considered undernourished if their body mass index (BMI) is below 18.5 kg/m^2. Individuals ages 5–19 years (61–228 months) are considered undernourished if their age-specific BMI values are below minus two standard deviations from the median of the reference population (https://www.who.int/growthref/en/). In the majority of countries, BMI-for-age covered individuals ages 15–19 years, as anthropometric data were available only for this age group; if other data were available, BMI-for-age was applied for all individuals ages 5–19 years. Children under age 5 years (under 60 months) are considered undernourished if their *z*-score for either height-for-age (stunting) or weight-for-age (underweight) is below minus two standard deviations from the median of the reference population (https://www.who.int/childgrowth/software/en/). Nutritional information is not provided for households without members eligible for measurement; these households are assumed to be nondeprived in this indicator.	1/6
	Child mortality	Any child under age 18 has died in the five years preceding the survey. When a survey lacks information about the date of child deaths, deaths that occurred at any time are taken into account.a	1/6
Education	Years of schooling	No household member of “school entrance age + six years” or older has completed six years of schooling.b	1/6
	School attendance	Any school-age childc is not attending school up to the age at which he or she would complete class 8.	1/6
Standard of living	Electricity	The household has no electricity.d	1/18
	Sanitation	The household does not have access to improved sanitation (according to Sustainable Development Goal guidelines), or it is improved but shared with other households. A household is considered to have access to improved sanitation if it has some type of flush toilet or latrine or ventilated improved pit or composting toilet that is not shared. When a survey uses a different definition of adequate sanitation, the survey report is followed.	1/18
	Drinking water	The household does not have access to an improved source of drinking water (according to Sustainable Development Goal guidelines), or an improved source of drinking water is at least a 30-minute walk from home, roundtrip. A household is considered to have access to an improved source of drinking water if the source is piped water, a public tap, a borehole or pump, a protected well, a protected spring or rainwater. When a survey uses a different definition of improved source of drinking water, the survey report is followed.	1/18
	Housing	At least one of the household’s three dwelling elements—floor, walls or roof—is made of inadequate materials—that is, the floor is made of natural materials and/or the walls and/or the roof are made of natural or rudimentary materials. The floor is made of natural materials such as mud, clay, earth, sand or dung; the dwelling has no roof or walls; the roof or walls are constructed using natural materials such as cane, palm, trunks, sod, mud, dirt, grass, reeds, thatch, bamboo or sticks or rudimentary materials such as carton, plastic or polythene sheeting, bamboo or stone with mud, loosely packed stones, uncovered adobe, raw or reused wood, plywood, cardboard, unburnt brick, or canvas or tent.	1/18
	Cooking fuel	The household cooks with dung, wood, charcoal or coal.	1/18
	Assets	The household does not own a car or truck and does not own more than one of the following assets: radio, television, telephone, computer, animal cart, bicycle, motorbike or refrigerator.e	1/18

a. Information about child deaths is typically reported by women ages 15–49. When information from an eligible woman was not available, information from a man was used when the man reported no death in the household, and information was coded as missing when the man reported a death (because the date of the death was unknown).
b. The cutoff was previously age 10, but this did not account for the fact that children do not normally complete six years of schooling by age 10. If a child starts school at age 6 and has completed six years of schooling by age 10, this is counted as an achievement. If not, this is not counted as a deprivation. This adjustment, which is conceptually better but minimally affects empirical estimates, applies only to countries with an updated survey in 2020.
c. Official school entrance age is from UIS.Stat (http://data.uis.unesco.org).
d. For the few countries that do not collect data on electricity because of 100 percent coverage, all households in the country as identified as nondeprived in electricity.
e. Televisions include smart televisions and black and white televisions, telephones include cell phones, computers include tablets and laptops, and refrigerators include freezers.

The MPI value is the product of two measures: the multidimensional poverty headcount ratio and the intensity of poverty:

$$MPI = H \cdot A$$

The contribution of dimension d to multidimensional poverty can be expressed as

$$Contrib_d = \frac{\sum_{j \in d} \sum_1^q c_{ij}}{n} / MPI$$

where d is health, education or standard of living.

The MPI can also be expressed as the weighted sum of the censored headcount rates h_j of each indicator j. The censored headcount rate of indicator j refers to the proportion of people who are multidimensionally poor and deprived in this indicator.

$$MPI = \sum_{j=1}^{10} c_j \cdot h_j$$

where c_j is the weight associated with indicator j (either 1/6 or 1/18), and the weights sum to 1.

The variance of deprivation scores of multidimensionally poor people is used to measure inequality among those people:

$$V = \sum_1^q (s_i - A)^2 / (q - 1)$$

All parameters defined above are estimated using survey data and sampling weights according to the rules of the sampling theory (Lohr 2010).

Weighted deprivations:

- Household 1: $(1 \cdot 16.67) + (1 \cdot 5.56) = 22.2$ percent.
- Household 2: 72.2 percent.
- Household 3: 38.9 percent.
- Household 4: 50.0 percent.

Based on this hypothetical population of four households:

Headcount ratio (H) =

$$\left(\frac{0+7+5+4}{4+7+5+4}\right) = 0.80$$

(80 percent of people are multidimensionally poor).

Intensity of poverty (A) =

$$\frac{(72.2 \cdot 7) + (38.9 \cdot 5) + (50.0 \cdot 4)}{(7+5+4)} = 56.3 \text{ percent}$$

(the average multidimensionally poor person is deprived in 56.3 percent of the weighted indicators).

$$MPI = H \cdot A = 0.8 \cdot 0.563 = 0.450.$$

Contribution of deprivations in:

Health:

$$contrib_1 = \frac{16.67 \cdot 5 + 16.67 \cdot (7+4)}{4+7+5+4} \Big/ 0.450 = 29.6\%$$

Education:

$$contrib_2 = \frac{16.67 \cdot (7+4) + 16.67 \cdot 7}{4+7+5+4} \Big/ 0.450 = 33.3\%$$

Standard of living:

$$contrib_3 = \frac{5.56 \cdot (7 \cdot 4 + 5 \cdot 4 + 4 \cdot 3)}{4+7+5+4} \Big/ 0.450 = 37.1\%.$$

Example using hypothetical data

Indicator	Indicator weight	Household 1	Household 2	Household 3	Household 4
Household size		4	7	5	4
Health					
At least one member is undernourished	(⅓) ÷ 2 = 16.7%	0	0	1	0
One or more children have died	(⅓) ÷ 2 = 16.7%	1	1	0	1
Education					
No one has completed six years of schooling	(⅓) ÷ 2 = 16.7%	0	1	0	1
At least one school-age child not enrolled in school	(⅓) ÷ 2 = 16.7%	0	1	0	0
Standard of living					
No electricity	(⅓) ÷ 6 = 5.6%	0	1	1	1
No access to improved sanitation	(⅓) ÷ 6 = 5.6%	0	0	1	0
No access to an improved source of drinking water	(⅓) ÷ 6 = 5.6%	0	1	1	0
House built with inadequate materials	(⅓) ÷ 6 = 5.6%	0	0	0	0
Household cooks with dung, wood, charcoal or coal	(⅓) ÷ 6 = 5.6%	1	1	1	1
Household does not own a car or truck and does not own more than one of the following assets: radio, television, telephone, computer, animal cart, bicycle, motorbike or refrigerator.	(⅓) ÷ 6 = 5.6%	0	1	0	1
Results					
Individual deprivation score, c (sum of each deprivation multiplied by its weight) for each household member		22.2%	72.2%	38.9%	50.0%
Is the household multidimensionally poor ($c \geq ⅓$)?		No	Yes	Yes	Yes

Note: 1 indicates deprivation in the indicator; 0 indicates nondeprivation.

Calculating the contribution of each dimension to multidimensional poverty provides information that can be useful for revealing a country's deprivation structure and can help with policy targeting.

Variance of deprivation scores among the poor (V) =

$$\frac{(0.722-0.563)^2 \cdot 7 + (0.389-0.563)^2 \cdot 5 + (0.500-0.563)^2 \cdot 4}{16-1}$$

$$= 0.023$$

Censored headcount rates for each indicator

	People who are multidimensionally poor and deprived in each indicator	Proportion of people who are multidimensionally poor and deprived in each indicator	Proportion of people who are multidimensionally poor and deprived in each indicator multiplied by the indicator weight
Nutrition	5	0.250	0.042
Child mortality	11	0.550	0.092
Years of schooling	11	0.550	0.092
School attendance	7	0.350	0.058
Electricity	16	0.800	0.044
Sanitation	5	0.250	0.014
Drinking water	12	0.600	0.033
Housing	0	0.000	0.000
Cooking fuel	16	0.800	0.044
Assets	11	0.550	0.031
Sum			0.450

Technical note 6. Planetary pressures–adjusted Human Development Index

The Planetary pressures–adjusted Human Development Index (PHDI) adjusts the Human Development Index (HDI) for planetary pressures in the Anthropocene. The PHDI discounts the HDI for pressures on the planet to reflect a concern for intergenerational inequality, similar to the Inequality-adjusted HDI adjustment, which is motivated by a concern for intragenerational inequality. The PHDI is computed as the product of the HDI and (1 – index of planetary pressures), where (1 – index of planetary pressures) can be seen as an adjustment factor.

The PHDI is the level of human development adjusted by carbon dioxide emissions per capita (production) and material footprint per capita to account for the excessive human pressure on the planet. It should be seen as an incentive for transformation. In an ideal scenario, with no pressures on the planet, the PHDI equals the HDI. However, as pressures increase, the PHDI falls below the HDI. In this sense the PHDI measures the level of human development when planetary pressures are considered.

Indicator definitions and data sources

In addition to the data used to calculate the HDI, the PHDI uses data on carbon dioxide emissions per capita (production) and material footprint per capita.

- Carbon dioxide emissions per capita (production): carbon dioxide emissions produced as a consequence of human activities (use of coal, oil and gas for combustion and industrial processes, gas flaring and cement manufacture), divided by midyear population. Values are territorial emissions, meaning that emissions are attributed to the country in which they physically occur. Data are from Global Carbon Project (2022).
- Material footprint per capita: material footprint is the attribution of global material extraction to domestic final demand of a country. Material footprint is calculated as raw material equivalent of imports plus domestic extraction minus raw material equivalents of exports. The total material footprint is the sum of the material footprint for biomass, fossil fuels, metal ores and nonmetal ores. Material footprint per capita describes the average material use for final demand. Data are from UNEP (2022).

Steps to calculate Planetary pressures-adjusted Human Development Index values

There are three steps to calculating PHDI values.

Step 1. Calculating the carbon dioxide emissions index and the material footprint index

Carbon dioxide emissions per capita and material footprint per capita are normalized in the same way as the components of the HDI. Through a min-max transformation each becomes an index with values between 0 and 1 calculated as:

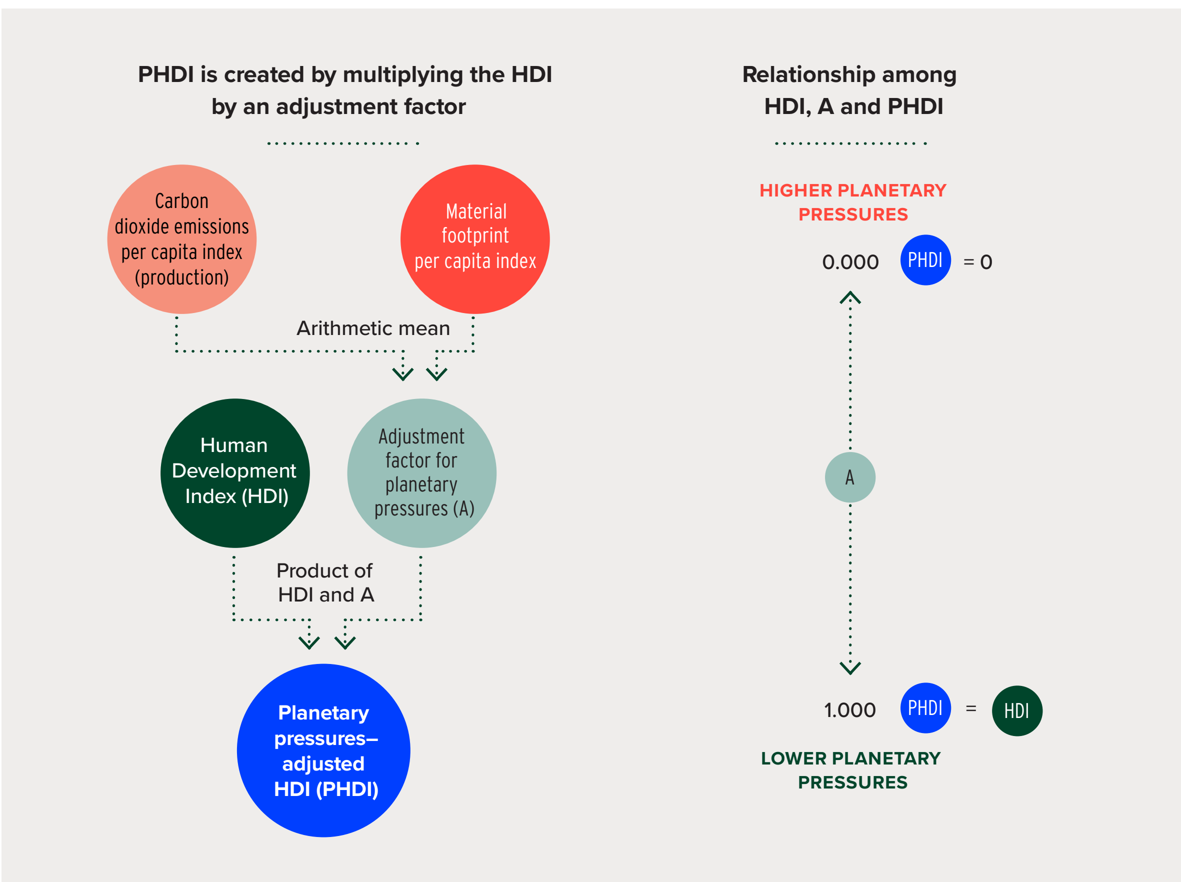
$$A_j \text{ index} = (\text{maximum}_j - \text{observed value}_j) / (\text{maximum}_j - \text{minimum}_j)$$

where $j = 1,2$ refers to the two included planetary pressure indicators.

Zero was set as minimum. The maximum corresponds to the maximum value observed historically for all countries since 1990, in line with the similar approaches in the literature, such as Biggeri and Mauro (2018). For carbon dioxide emissions per capita the maximum value is 68.72 tonnes, observed for Qatar in 1997. For material footprint per capita the maximum value is 107.42, observed for Kuwait in 1996. The ranking of countries is sensitive to the selection of the maximum.

For both carbon dioxide emissions per capita and material footprint per capita, the higher the observed value and the closer to the maximum, the higher the pressures on the planet, implying a smaller value of the index and a larger adjustment to the HDI.

Figure 1 Relationship of the components of the Human Development Index and the Planetary pressures–adjusted Human Development Index



Source: Human Development Report Office.

Step 2. Constructing the adjustment for planetary pressures

The adjustment factor for planetary pressures (A) is the arithmetic average of the indices measuring carbon dioxide emissions per capita and material footprint per capita, which assumes perfect substitution of these two indicators. Lower pressures on the planet result in a larger A and smaller adjustment to the HDI (see figure 1).

$$A = (\text{Carbon dioxide emissions index} + \text{material footprint index}) / 2$$

In addition, the index of planetary pressures, P, is defined as the complement of A: $P = (1 - A)$.

Step 3. Adjusting the Human Development Index to account for planetary pressures

The PHDI is the product of the HDI and the adjustment factor, A:

$$PHDI = HDI \cdot A,$$

or, equivalently, $PHDI = HDI \cdot (1 - P)$.

The difference between the HDI and the PHDI values due to planetary pressures, expressed as a percentage, is:

$$\text{Difference in HDI value} = \left(\frac{HDI - PHDI}{HDI}\right) \cdot 100$$

$$= P \cdot 100$$

Example: Netherlands

Indicator	Value
Human Development Index (HDI)	0.941
Carbon dioxide emissions per capita (production), tonnes	8.1
Material footprint per capita, tonnes	32.3
Carbon dioxide emissions index	(68.72 – 8.1) / 68.72 = 0.883
Material footprint index	(107.42 – 32.3) / 107.42 = 0.700
Adjustment for planetary pressures factor (*A*)	(0.883 + 0.700) / 2 = 0.791
Planetary pressures–adjusted HDI (PHDI)	0.941 · 0.791 = 0.745
Difference in HDI value (%)	[(0.941 – 0.745) / 0.941] · 100 = 20.8

Note: Values are rounded.

Difference in Planetary pressures–adjusted Human Development Index rank and Human Development Index rank

Difference in ranks on the PHDI and the HDI is calculated only for countries for which both a PHDI and an HDI value are calculated. Due to data limitations, PHDI values are calculated for a smaller set of countries. To calculate the PHDI rank difference from the HDI rank, HDI ranks are recalculated for the set of countries for which a PHDI value is calculated.

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