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Brief on Discounting in the Context of Climate Change Economics

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I. BACKGROUND

The debate on the economics of climate change is often framed as a choice between two possible scenarios. In a stylized and ultra-simplified situation we can choose to live in a world where we do nothing now to mitigate climate change, but in the future will have to suffer income losses from the damages of warming—say, a loss in yearly GDP, one or two hundred years from now. Or, we can choose to live in a world where those future income losses are avoided, but at the expense of taking costly action starting now or very soon, which implies cuts in yearly GDP in the short-run. Naturally, in this second world, mitigation corresponds to future benefits in terms of avoided losses from climate change. But mitigation also has costs in terms of income foregone to pay for those future benefits.

This is how cost-benefit analysis of climate change is typically framed. This single world analysis relates to trade-offs in the distribution of income across generations. Why across generations? Because climate change seen from a single world perspective is a very long-term challenge. Emissions of greenhouse gases that drive warming are almost irreversible. Substantial amounts—up to a third—of today’s emissions will stay in the atmosphere for centuries. For example, emissions at the outset of the Industrial Revolution are still with us. Even if we were to stop all emissions now, past emissions would persist in the atmosphere and would affect climate well into the next century and beyond. The relationship between emissions, concentrations of greenhouse gases in the atmosphere, the radiative (or warming) effect of those concentrations, and finally the impact of the radiative effect on climate patterns are complex and develop typically over long periods of time.

There are two important implications of the fact that climate change is a very long-term problem. The first is that the choice of the discount rate—used to make comparisons of income losses across time—is critical to assess the trade-offs between the more immediate costs of mitigation and the longer term losses from warming. Given that discounting takes place over many years, even relatively small differences in the discount rate can have a very large impact on this assessment. There is some debate on whether the estimates of income losses from mitigation and warming are reasonable. Almost everybody agrees that the long-term losses from warming typically underestimate damages from climate change once natural assets that are not captured by national accounts—like the existence value for certain species or ecosystems—are factored in. But the effect of the discount rates typically—although not always, as will be explained later—overwhelms the effect of differences in costing.

The second implication of the long-term nature of climate change is high uncertainty. In a single-world analysis, the relevant timeframe is measured in centuries. We only need to have in mind how the world was in 1907 to realize how much can change over the course of a century. The long-term nature of climate change, coupled with the complexity of climate systems, implies that while we know that the Earth is warming, we don’t know how fast and how far the implications from this warming will be. Catastrophic outcomes that can change the world in fundamental ways—like a new ice age or increases in sea levels by twenty meters—while unlikely in the short run, are possible without mitigation

within a timeframe of one to two centuries. This implies, in turn, that a deterministic setting may be incomplete for analyzing climate change. Thus, a second dimension of the debate occurs in the context of explicitly considering risk and uncertainty when deciding on the values of income losses and of the discount rate, and what this implies in terms of the rationale and timing for mitigation.

But this simplified and ultra-stylized single world analysis is grossly incomplete as a framework to analyze the challenge of climate change. The world is not composed of a single individual or a group of identical individuals. It is important to take into account asymmetries that exist in the world such as income distribution across generations. The distribution of cross-generational income intersects with climate change at two levels: first, in terms of the *causes* of climate change (emissions of greenhouse gases) and second, in terms of the *effects* of climate change.

In terms of *causes* of climate change, differences across countries in the accumulation of global income over time are intimately linked to asymmetries in contributions to the current stocks of greenhouse gases. There isn't a one-to-one relationship between rich countries today and historical emissions, in part because some heavily industrialized countries have fallen back economically (like the transition economies). But today's richest countries have the overwhelming responsibility for the accumulated stocks of greenhouse gases in the atmosphere. If we look at today's emissions—that is, flows rather than stocks—rich countries remain important, but a few large developing countries are also starting to contribute significantly. There is a third group of countries that have not contributed either historically or now, and which have the lowest shares of today's world income.

Emission paths into the future will also have important income distribution implications. For example, strong mitigation measures may hurt economies that are heavily dependent on fossil fuels. Within countries, many people that work in or are dependent on emission-intensive activities may be economically displaced. The losses of income that are predicted if mitigation is taken-up will fall on some people more than others. Of course, these future income distributional issues linked with emissions can be managed, namely with transfers of income that can ensure a fair and equitable burden sharing, also taking into consideration the historical and present levels of emissions.

In terms of the *effects* of climate change, the distributional implications will be much more difficult to manage. When thinking about the distributional implications of the effects of climate change, it is really the future that matters most. While temperature has been increasing since the Industrial Revolution, global warming is only now starting to affect climate patterns. Some regions in the North may benefit from warmer temperatures and increase in rainfall. But the effects in tropical regions are and will be overwhelmingly negative. The poorest countries and people are suffering the most in terms of the effects of climate change, and this pattern is likely to persist in the future. If this pattern does persist, climate change is likely to drive income divergence and to deepen global inequalities.

The distribution of global income within generations—in the past, now and in the future—is equally critical. If we go beyond the single-world approach and take into account within generation income distribution implications of climate change, then the relative importance of losses of income from mitigation and warming are not the same for different regions and groups of people. If we exclude extreme consequences of warming, such as the catastrophic events that are likely to leave every country and virtually every person worse off, then the urge to act, the weight given to the future, the tolerance for risk, and other factors that determine how strongly and quickly we want to start addressing climate change will probably vary depending on people’s place of residence and income level.

There are many complex issues at stake and many perspectives from which to address these issues. Questions include how to estimate the costs of mitigation and the losses from warming for different countries. Other important questions relate to the adaptation measures that need to be taken in the short run to help the most vulnerable countries, and how to finance them.

The aim of this note is simply to focus on a specific dimension: it will review the debate on discounting considering three successive aspects. Section II takes the single-world perspective and explores in detail how the expression that determines the discount rate emerges in a simplified deterministic setting. It uses Ramsey’s model of “consumption smoothing” as the main approach. The discount rate under this approach depends on preference-related parameters and the rate of growth of the global economy. Section III extends the discussion on the discount rate to a stochastic setting, incorporating uncertainty in the model, but still under a single-world framework. Under this approach, the discount rate is not uniquely determined if we take account risk while the analysis unravels if we take a further step of incorporating structural uncertainty. Section IV discusses discounting when we go beyond a single world approach to an asymmetric world where heterogeneity in preferences and income distribution aspects matter and have to be taken into consideration. Finally, concluding remarks are found in section V.

II. DISCOUNTING UNDER CERTAINTY IN A SINGLE WORLD

There is a long-running debate in economics on how to determine an appropriate social discount rate. Both ethical judgments and empirical information that assess preference parameters from observed behavior are used—sometimes in combination. Under strong assumptions and simplifications, it is possible to express a global social discount rate through the expression below.¹ One possible way of deriving it is presented in the Appendix.

$$r = \rho + \eta \cdot g$$

This is the Ramsey formula for the social discount rate, where:

- r is the social discount rate and it is used to discount consumption.
- ρ is the pure time preference rate, and it discounts utility.
- η is the elasticity of marginal utility of consumption, measuring the relative curvature of the utility function—how quickly utility drops as consumption increases.
- g is the growth rate of consumption per capita.

How does discounting relate to the economic analysis of climate change? While the specific way in which discounting affects the outcomes of many of the models used in cost-benefit analysis is complex (in many cases unknown because many authors have not made sensitivity analysis) in general high social discount rates favor a “ramp-up” approach to mitigation, while low discount rates favor immediate action. In the context of the single-world analysis under certainty, the issue is one of “consumption smoothing,” and the debate is centered on the choices for the preference-related parameters of the discount rate: the rate of pure time preference (ρ) and the elasticity of marginal utility with respect to consumption (η). Table 1 provides a summary of choices of values by some of the leading authors, and some of the criticisms that have been weighed against these choices.

¹ These assumptions include: overall objective is assumed to be to maximize a social welfare function which is expressed as the integral across all households and all time of the utility of households; the utility is additively separable across time; the only determinant of utility is consumption at each instant; there is one representative household and the utility function is unchanging over time; utility is assumed to take a constant-elasticity-of-substitution functional form. See the Appendix for details.

Table 1: Discounting in the context of climate change

Author	ρ	Criticism	η	Criticism
Nordhaus (1994)	3%	ρ of 3% is unconscionable (Delong 2006)	1	η is too low to yield reasonable saving rates (Dasgupta 2007)
Cline (1992)	0- stewardship argument	<ul style="list-style-type: none"> ▪ ρ is too small to yield reasonable saving rates (Arrow 1995; Weitzman 2007) ▪ ρ and η are not independent and given $\eta=1$, ρ has to be higher to be consistent with Ramsey optimal growth model (Nordhaus 2006) ▪ ρ also needs to be higher to match observed market data (Nordhaus 2006) ▪ ρ is too low relative to revealed aversion to risk in insurance data (Gollier 2006) 	1.5	
Stern (2006)	0.1%, standard utilitarian view and hazard rate argument		1	
Dasgupta (2007)	0		[2,4], inequality aversion and risk aversion argument	Dasgupta's analysis on η omits technical progress which could imply much lower saving rates. (Delong 2006; Dietz 2007)

Source: Own elaboration

The rate of pure time preference, ρ , has several possible interpretations. From an individual perspective, ρ measures “impatience”—since people in general prefer deriving utility today rather than tomorrow, it is generally assumed that the rate of pure time preference is positive. The higher the value of ρ , the more we discount the future.

The intertemporal allocation seen from an individual perspective has only limited relevance for the cost benefit analysis of climate change. What is at stake is a social discount rate, in which ρ measures the importance of the utility that we attribute today to the welfare of future generations—it has an ethical meaning. A zero ρ means that the welfare of future generations is treated equally to the one of present generations. A positive ρ means that the utility (or welfare) of future generations are reduced or “discounted” compared to present generations. The higher the value of ρ , the more present generations discount future generations’ welfare.

A stewardship argument—“we should do not discriminate against future generations just because they haven’t yet been born”—would only justify a non-zero rate of pure time preference in terms of the probability that the human race will be extinct. This principle of universalizability (i.e. all generations should be treated alike) is proposed by some economists such as Pigou (1932), Ramsey (1928), Harrod (1948), Koopmans (1965), Solow (1974) and Stern (2006) to argue for a low ρ close to zero. Stern’s value of 0.1 is derived from the probability of human extinction given some catastrophic event. However, other authors like Arrow (1995) and Nordhaus (2007) argue that the

implications of a close-to-zero rate of pure time preference rate in standard savings and investment models is inconsistent with plausible and observed savings behavior.

The elasticity of marginal utility of consumption η measures the percentage change in marginal utility derived from one percentage change in consumption. Dasgupta (2007) interprets η as the elasticity of marginal well-being. In the general conditions of the Ramsey equation, $1/\eta$, measures the intertemporal elasticity of substitution while its reciprocal, η , also measures the coefficient of relative risk aversion.

The interpretation of η is difficult, since it can represent three concepts of aversion at once. First, η can be interpreted as a measure of *personal risk aversion* towards future consumption fluctuations. Second, η can be interpreted as a measure of *aversion towards inter-generational inequality*. A low η indicates a low degree of aversion towards inter-generational inequality. The intuition is that with a low η , and with a positive growth rate, the relatively-poor present generations care less about inter-generational inequality caused by redistributing income across time from the present poor to the future rich. The higher the η , the faster marginal utility diminishes, and the consumption becomes worth more to the present poor relative to the future rich. If the present poor are concerned about inter-generational inequality and want to avoid the income redistribution from the present poor to the future rich, a higher η should be chosen. In other words, if we are to smooth inter-generational consumption, larger values of η should be admitted. According to the Ramsey formula, a higher η leads to a higher discount rate r . The implication is that if future generations are going to be better off (with more to consume) than present generations, then our willingness to sacrifice on their behalf is certainly reduced. Third, η can also be interpreted as a measure of *aversion towards intra-generational inequality* (i.e. inequality across space).

Values for the coefficient of relative risk aversion η are commonly taken to be somewhere between one and four. The Stern Review (2006) chooses a value of one. As Nordhaus (2007) points out, in calibrating a growth model, the pure time preference rate ρ and the elasticity of marginal utility of consumption η can not be chosen independently because r and g are given, so this determines the values of ρ and η . A low η implies a relatively high ρ , and a high η implies a relatively low ρ , if the Ramsey formula is used to calibrate using observed interest and growth rates. For example, if r is 10% and the growth rate g is assumed to be 3%, then if we choose a low value of η —let's say of 1—this implies a high value of ρ at 7%. If we choose a high η —let's say of 3—this implies a low value of ρ at 1%. However, some authors question the validity of these calibrations—and moreover note that Nordhaus still only has one equation to determine the value of two parameters, so at least one must be chosen outside the empirical calibration framework.

III. DISCOUNTING UNDER UNCERTAINTY IN A SINGLE WORLD

As scene in the previous section, the social discount rate is equal to a unique interest rate under the simple deterministic Ramsey model. However, the analysis of discounting under *uncertainty* forces us to recognize that there is no single discount rate, in the same way that there is no single “rate of interest” in the economy.

There are two issues to tackle under a framework of uncertainty. The first discusses discounting *under risk*, that is, when the parameters describing the probabilistic behavior of g are assumed to be known with certainty. The second looks at the consequences of *structural uncertainty*, that is, what happens when the parameters describing the probabilistic distribution of g are, themselves, unknown.

a. Discounting Under Risk

The analysis of discounting under *risk* forces us to recognize that there is no single “rate of interest” in the economy, because the remuneration rate of assets depends on how risky they are. In the analysis, all the uncertainty is captured in the path of consumption per capita—or in its growth rate, g .

Weitzman (1999) shows that uncertainty reflected in future discount rates will result in a declining certainty-equivalent social discount rate over time. Interest rates under uncertainty do not aggregate arithmetically into a certainty-equivalent interest rate. Instead we should average the probabilistic discount factors, not the likely future discount rates. If there is a subjective probability p_i that future discount rate r_i is the correct rate to use, then the effective discount rate for time t will be:

$$r(t) = -\frac{\log \sum p_i e^{-r_i t}}{t}.$$

This effective discount rate declines monotonically over time from the expected interest rate $r(0) = \sum p_i r_i$ to an asymptotic limit of $r(\infty) = \min \{r_i\}$.

Table 2 shows a numerical example to show that this averaging process produces discount rates that decline with time. In the limit, as t goes to infinity, the discount rate converges to the lowest possible discount rate. Similar to Weitzman (1999), we assume 10 potential future discount rate scenarios, and each scenario has an equal probability: $p_1=p_2=\dots=p_{10}=0.1$.

Table 2: Numerical example of Weitzman's declining certainty-equivalent discount rate

Interest rate scenarios	discount factors in period t							
	10	50	100	200	500	1000	2000	5000
1%	0.90	0.61	0.37	0.14	0.01	0.00	0.00	0.00
2%	0.82	0.37	0.14	0.02	0.00	0.00	0.00	0.00
3%	0.74	0.22	0.05	0.00	0.00	0.00	0.00	0.00
4%	0.67	0.14	0.02	0.00	0.00	0.00	0.00	0.00
5%	0.61	0.08	0.01	0.00	0.00	0.00	0.00	0.00
6%	0.55	0.05	0.00	0.00	0.00	0.00	0.00	0.00
7%	0.50	0.03	0.00	0.00	0.00	0.00	0.00	0.00
8%	0.45	0.02	0.00	0.00	0.00	0.00	0.00	0.00
9%	0.41	0.01	0.00	0.00	0.00	0.00	0.00	0.00
10%	0.37	0.01	0.00	0.00	0.00	0.00	0.00	0.00
Certainty-equivalent discount factor	0.60	0.15	0.06	0.02	0.00	0.00	0.00	0.00
Certainty-equivalent discount rate	5.09	3.75	2.84	2.08	1.46	1.23	1.12	1.05

Source: own calculations.

Weitzman (2007) goes one step further and formally models two distinct interest rates by making the growth rate of consumption per capita g a random variable. He argues that all the uncertainty is captured in the path of consumption growth per capita g . Such an uncertainty will lead to a lower social discount rate than the one derived from the simple deterministic Ramsey model.

The extension of the Ramsey model to a setting under uncertainty where the growth rate “ g ” is assumed to be normally distributed with mean μ and standard deviation σ (both known with certainty)

$$g \sim N(\mu, \sigma)$$

can be summarized with the expression that provides the expected marginal utility (EMU) of one additional sure unit of consumption:

$$EMU = e^{-\rho - \eta\mu + \frac{1}{2}\eta^2\sigma^2}$$

Therefore, the (risk-free) interest rate is given by the stochastic Ramsey formula:

$$r^f = \rho + \eta\mu - \frac{1}{2}\eta^2\sigma^2$$

Thus, uncertainty will *always* bring the risk-free rate down compared with the interest rate under the deterministic Ramsey formula (if one assumes that g in the determinist setting equals the mean μ of the probabilistic setting). The higher the uncertainty as measured by σ , the more the risk-free interest rate comes down.

But, as noted, with uncertainty riskier assets have interest rates that differ from the risk-free interest rate. For example, the remuneration of equity (riskier) assets can be shown to be theoretically given by:

$$r^e = \rho + \eta\mu - \frac{1}{2}\eta^2\sigma^2 + \eta\sigma^2$$

Thus, equity holders require an additional remuneration for holding risky assets, captured by the term $\eta\sigma^2$, which is a measure of the equity risk premium. In comparison with the deterministic Ramsey formula, re-writing the equity interest rate as:

$$r^e = \rho + \eta\mu + \eta\left(1 - \frac{\eta}{2}\right)\sigma^2$$

shows that for values of η between 1 and 2 the equity interest rate will always be higher than the determinist Ramsey interest rate.

What rate, then to use? In a more general framework, the interest rates used to discount at time t for investment in assets with risk “in-between” the risk-free rate and the (risky) equity rate is given by (log) weighted average of the two interest rates:

$$r(t) = -\frac{\log\left[\beta e^{-r^e t} + (1-\beta)e^{-r^f t}\right]}{t}$$

where β stands for the “quantity of risk” in the context of the capital asset price model, measuring the systematic correlation between the returns to the asset and the equity market returns. If β is zero, then there is no risk so the rate becomes the risk-free rate r^f , and if β is one, then the asset is as risky as equity, so the rate becomes r^e . The expression can be re-arranged and expressed as:

$$r(t) = r^f - \frac{\log\left[\beta\left(e^{-\eta\sigma^2 t} - 1\right) + 1\right]}{t}$$

The above expression shows that the discount rate is an additive function of the risk-free interest rate and a term that depends on β and on the equity premium. Thus, there is no “unique” interest rate to use for discounting, as the deterministic model suggests.

Gollier (2002) treats uncertainty over consumption growth g in a different way. He expresses the discount rate as a sum of three components: the rate of pure time preference (ρ), the pure wealth effect (defined as the expected change in consumption times the absolute coefficient of risk aversion), and the prudence effect (defined as a function of the variance of change in consumption, the absolute coefficient of risk aversion and the coefficient of absolute prudence).

$$r = \rho + E(\Delta c) \cdot \left(-\frac{u''(c)}{u'(c)}\right) - \frac{1}{2} \text{Var}(\Delta c) \left(-\frac{u''(c)}{u'(c)}\right) \left(-\frac{u'''(c)}{u''(c)}\right)$$

The first term in the above equation of the discount rate r is the rate of pure time preference ρ ; the second term is the pure wealth effect which is positive if consumption is expected to grow (i.e. $E(\Delta c) > 0$) and individuals are risk averse to consumption fluctuations over time (i.e. $-\frac{u''(c)}{u'(c)} > 0$); the third term is the prudence effect which captures how uncertainty over consumption growth g could affect the discount rate r . The third term will have an unambiguous negative impact on r if individuals exhibit absolute risk aversion (i.e. $-\frac{u''(c)}{u'(c)} > 0$) and absolute prudence² (i.e. $-\frac{u'''(c)}{u''(c)} > 0$).

In Gollier's approach, the wealth effect and the prudence effect act in opposition to one another in determining the discount rate. When individuals in the economy are prudent (that is, their response to uncertainty is to save more), the wealth effect is offset, and the optimal discount rate is lowered.

To compare Gollier (2002) with Weitzman (2007), we find that both papers argue for a lower discount rate due to uncertainty over consumption growth but they provide different reasoning behind such a decline in the discount rate.

In Weitzman (2007), the risk-free discount rate looks like:

$$r^f = \rho + \eta\mu - \frac{1}{2}\eta^2\sigma^2,$$

In Gollier (2002) if we assume $\frac{\Delta c}{c} = g \sim N(\mu, \sigma)$, Gollier's expression for the discount rate becomes:

$$r = \rho + \eta\mu - \frac{1}{2}\sigma^2(\eta) \left(-\frac{u'''(c)}{u''(c)}\right) \cdot c$$

Therefore, Weitzman (2007) shows that uncertainty will *always* bring the risk-free discount rate down compared with the interest rate under the deterministic Ramsey formula because the third component is unambiguous negative. Gollier (2002) also shows

² The Kimball's coefficient of absolute prudence is defined as $-\frac{u'''(c)}{u''(c)}$, and the Arrow-Pratt coefficient of absolute risk aversion is defined as $-\frac{u''(c)}{u'(c)}$. Positive prudence is defined as $-\frac{u'''(c)}{u''(c)} > 0$.

uncertainty will bring the discount rate down as long as individuals exhibit absolute risk aversion and absolute prudence³.

b. “Discounting” Under Structural Uncertainty⁴

A further complication on determining the interest rate is if the parameters μ and σ (that described the distribution of growth) are unknown. This type of analysis is relevant, for instance, when there is a small (non-zero) probability of a major catastrophic event. Intuitively, this means that the probability distribution of growth has a fat (left) tail.

One way of “modeling” this formally corresponds to sorting out the implications of what happens if the uncertainty of σ is explicitly incorporated in the analysis. We can either estimate it from past observations or from a simulation. Then the estimate $\hat{\sigma}$ is itself distributed following a t-student distribution. The implication of explicitly recognizing the uncertainty of σ is that in this case, due to the fat tail of the t-student distribution, the expected marginal utility of an additional sure unit of consumption is unbounded:

$$EMU \rightarrow \infty$$

The structural uncertainty represented by explicitly incorporating our ignorance about the true values of the parameters defining the distribution of growth completely overwhelms any effect (and choices) we can make about the discount rates and the parameters that determine it—regardless of whether we are under a deterministic or a stochastic setting.

The interpretation is that the conventional “consumption smoothing” analysis developed using marginal reasoning is no longer meaningful. The interpretation has to move towards justifications for catastrophe insurance as the rational for making resource allocations today to “thin” the tails of the distribution of growth to avoid extreme outcomes in the future.

³ Decreasing absolute risk aversion will guarantee the growth uncertainty affects the discount rate negatively. Decreasing absolute risk aversion is defined as that its marginal absolute risk aversion is negative, i.e. $d(-\frac{u''(c)}{u'(c)})/dc < 0$ which implies $-\frac{u'''(c)}{u''(c)} > -\frac{u''(c)}{u'(c)} > 0$. The empirical evidence on decreasing absolute risk aversion is mixed.

⁴ This section draws mainly on Weitzman (2007).

IV. DISCOUNTING IN AN ASYMMETRIC WORLD

The discussion so far has not considered the implications of asymmetries in preferences, the distribution of income, or other factors when it comes to determining a global social discount rate. We have assumed that the world could be represented by just one individual or a group of identical individuals at each point in time. But in reality, the world is not composed of a single individual or a group of identical individuals, and the impact of climate change for different countries and communities will be very diverse. Thus it is important to take inter-generational and intra-generational inequality into account and see how factoring in the asymmetries that exist in the world affect the determination of global social discount rate.⁵ We are especially interested exploring how the relaxation of homogeneity assumption in the simple Ramsey growth model will change the discount rate used in the economics of climate change.

a. Accounting for Inter-Generational Inequality

Inter-generational inequality raises the question whether a higher or a lower discount rate should be used in the economics of climate change.

On the one hand, impacts of climate change on future generations raise very firmly questions of ethics. It has been argued that inter-generational inequity concerns should be incorporated into climate risk assessments by applying a lower “ethical” rate of pure time preference ρ to evaluate benefits received by future generations, so as to not trivialize these benefits relative to current costs. Pigou (1932), Ramsey (1928), Harrod (1948), Koopmans (1965), Solow (1974) and Stern (2006) all argue for a low ρ close to zero.

On the other hand, the concern about inter-generational inequality will argue for a higher η and a higher discount rate r . Given positive economic growth, future generations are going to be better off (with more to consume) than present generations. Thus inter-generational inequality will enlarge if present generations sacrifice on their behalf for future generations. Thus as Dasgupta (2007) argues, if we are to avoid the income redistribution from the present poor to the future rich and smooth inter-generational consumption, larger values of η should be admitted.

But a third parameter in the Ramsey formula—the growth rate of consumption per capita “ g ”—is often omitted from the important discussion of inter-generational inequality issues of climate change. We argue that a different approach should be used to measure the growth rate so as to capture how much better off future generations are going to be than present generations (in other words, how big inter-generational inequality is). This will lead to a lower discount rate used in the economics of climate change.

⁵ Stern (2007) also points out that standard treatments of discounting are valuable for analyzing marginal projects but are inappropriate for non-marginal comparisons of paths. The approach to discounting must meet the challenge of assessing and comparing paths that have very different trajectories and involve very long-term and large inter-generational impacts. We must go back to the first principles from which the standard marginal results are derived.

In the simple Ramsey growth model, g represents the reduced-form representation of “technology” in a Solow model. In a model without technological change where growth comes only from capital accumulation with decreasing returns to capital, the steady state growth rate is zero. From the Ramsey formula, we see that a low pure time preference rate ρ and/or a low elasticity of marginal utility of consumption η does not necessarily imply a low discount rate r . Growing consumption g can be a reason for discounting. If future generations are going to be better off (with more to consume) than present generations, then our willingness to sacrifice on their behalf is certainly reduced. The higher g implies that it would require a greater discount rate to justify our depriving ourselves of consumption. Similarly if consumption were falling the discount rate would be negative.

However, if we use another methodology to measure global growth rates, the discount rate could be reduced. The growth rate of consumption per capita g is usually calculated using the conventional per capita “GDP-weighted” growth rate. The GDP-weighted growth rates are perhaps the most often cited ones due to their ready availability in all databases. This method is especially appropriate for monitoring regional or global GDP growth. But the GDP-weighted growth could be inappropriate for other purposes. For example, in the context of Africa, where South Africa and Nigeria account for 50 percent of the total regional output, the GDP-weighted growth at the regional level reflects mainly the growth performances of these two countries. Therefore, some researchers prefer the “population-weighted” GDP growth rates, which they argue representative of the growth experienced by a typical African (Collier and O’Connell 2005).

So if we measure the growth rate of consumption per capita using population as weights, we probably will see a higher global growth rate of consumption per capita in recent years (mainly due to the high economic growth rates of China and India). On the other hand, it also implies that the future negative effect of global warming on the growth of those fast-growing populous developing economies will reduce the overall global growth rate if we use the methodology of “population-weighted” growth rates. So when we simulate the future global growth rate with the threat of climate change, the potential growth rate of consumption per capita will be lower than has been suggested in the literature. Thus it could lead to a lower discount rate.

b. Accounting for Intra-Generational Inequality

Apart from *inter-generational* inequality, several extensions will capture *intra-generational* inequality (or more generally, heterogeneity) in the discounting analysis of climate change. All these extensions will argue for a lower discount rate. Among these extensions, two capture heterogeneity in the rate of pure time preference ρ , while another two capture modifications in the elasticity of marginal utility of consumption η .

Heterogeneity in the rate of pure time preference ρ

Given that people have different rates of pure time preference ρ , Gollier and Zeckhauser (2005) provide a model that captures heterogeneous time preferences and analyze the effect of such heterogeneity on aggregate discount rate. In their model, the rate of pure time preference is heterogeneous across the population (i.e. the rate of pure time preference ρ_i is individual specific). Wealthier individuals are more patient and tend to have a lower rate of pure time preference ρ_i .

Gollier and Zeckhauser (2005) aim to find the collective rate of pure time preference $\rho(t)$ at time t for the representative agent whose behavior duplicates the behavior of the whole population⁶. The collective rate of pure time preference $\rho(t)$ is a weighted mean of the members' individual rates of pure time preference ρ_i . Each member's weight is proportional to his/her degree of absolute risk aversion for composition fluctuations $\frac{u_i''(c(t))}{u_i'(c(t))}$.

The collective rate of pure time preference $\rho(t)$ is:

$$\rho(t) = \frac{\sum \rho_i * \left(-\frac{u_i''(c(t))}{u_i'(c(t))}\right)}{\sum \left(-\frac{u_i''(c(t))}{u_i'(c(t))}\right)}.$$

It is important to notice that the weights $-\frac{u_i''(c(t))}{u_i'(c(t))}$ in the above equation are a function of time t . Thus even if the individual rate of pure time preference ρ_i is constant over time, the representative agent's rate of pure time preference is time-varying. Furthermore, this aggregation rule implies that the collective rate of pure time preference $\rho(t)$ is decreasing with respect to the time horizon when wealthier consumers are less risk averse to consumption fluctuations, a common assumption (with a smaller $-\frac{u_i''(c(t))}{u_i'(c(t))}$).

Gollier and Zeckhauser (2005) thus conclude that the representative agent will have a declining discount rate when individuals have decreasing risk aversion preferences even though all members of a group have a constant discount rate.

Li and Lofgren (2000) apply the social-choice approach and look at a similar model where two different rates of pure time preference exist in the society. One is held by utilitarians, and the other held by conservationists. Utilitarians' rate of pure time

⁶ It is a common practice in macroeconomics to use a representative agent to represent the whole group although aggregation of individuals' heterogeneous behavior is always a concern. In the classic case with homogeneous time preference, the collective rate of pure time preference for the representative agent is not a function of time t . But with heterogeneous time preferences, time enters as an additional factor.

preference is higher than conservationists'. Li and Lofgren (2000) show that a declining ρ is consistent with a rule whereby current (future) generations must always take into account the well-being of future (current) generations.

Modifications to the elasticity of marginal utility of consumption η

Intra-generational inequality issues can be also captured using the elasticity of marginal utility of consumption η . For example, Dasgupta (2007) interprets the parameter η as a measure of aversion towards consumption inequality among people. According to Dasgupta (2007), not only inter-generational inequality matters in discounting but also does intra-generational inequality. Climate change is predicted to inflict far more damage to the tropics (the poor world) than to the temperate zone (the rich world). If present generations are concerned about intra-generational inequality, a higher η should be used. As climate change will have a disproportionate negative impact on the world poor (whose marginal utility is high because η is implicitly large) urgent action is needed to avoid the increase in inequality between the poor and the rich. In this case, a higher η will lower the discount rate, which is not captured in the simple Ramsey model. Dasgupta (2007) recommends increasing η to 3 to reduce intra-generational inequality.

Atkinson and Brandolini (2006) argue that η shouldn't be a constant. Instead η should first rise as income rises, then fall. The reason is that a low or zero elasticity η indicates our interest in redistributing income below a poverty line. But we would or should not be interested in redistributing income amongst the rich.

A more straightforward way to tackle intra-generational inequalities across income groups (individuals or regions) is to write a group-specific utility function of consumption c_i at each point in time. We then add utility across groups in that generation and integrate utility over time.

Suppose c_i is consumption per capita for group i , and the group-specific discount factor λ_i can be expressed as:

$$\lambda_i = u'(c_i)e^{-\rho t}$$

The aggregate discount factor λ can be expressed as:

$$\lambda = \sum_{i=1}^N \alpha_i u'(c_i) e^{-\rho t}$$

where N is the number of income groups or regions and α_i describes how a unit increment in aggregate consumption is distributed across groups.

If the increment is distributed equally across groups, the aggregate discount factor λ becomes:

$$\lambda = \sum_{i=1}^N (1/N) u'(c_i) e^{-\rho t}$$

If we look at the above equation of λ as an expected welfare function for utility taking the functional form of u' , we can use the Atkinson theorem⁷ to analyze how an enlarging intra-generational inequality will affect the aggregate discount factor λ and then the discount rate r .

Atkinson (1970) establishes a well-known theorem covering the passage from inequality to welfare. He proves that if a decision-maker is income-seeking and inequality-averse, Lorenz-dominance is a necessary and sufficient condition to detect welfare superiority in the dominating distribution, provided that the dominating distribution has the same or higher mean than that of the dominated distribution.

This result of welfare ranking of different distribution is important because it is independent of the exact functional form of the welfare function. The result only requires that the welfare/utility function increases in income/consumption and that the welfare/utility function is concave (inequality averse). In simpler words, the Atkinson theorem concludes that an enlarging unequal distribution will lower the welfare if the welfare/utility function increases in income/consumption and is concave. If the welfare/utility function decreases in income/consumption and is convex, the Atkinson theorem concludes the opposite: an enlarging inequality will increase welfare instead.

Applying the Atkinson theorem to the above aggregate discount factor equation to compare two discount factors λ and λ^{new} with different consumption profiles c_i and c_i^{new} within a generation, we obtain:

$$\lambda = \sum_{i=1}^N (1/N) u'(c_i) e^{-\rho t}$$

$$\lambda^{new} = \sum_{i=1}^N (1/N) u'(c_i^{new}) e^{-\rho t}$$

Assume c_i^{new} is more unequal than c_i , if $u'(c_i)$ decreases in c_i and is convex⁸, Atkinson's theorem will conclude that $\lambda^{new} > \lambda$ and the corresponding discount rate $r^{new} < r$. Thus an enlarging intra-generational inequality leads to a lower discount rate and justifies an urgent action to tackle global warming.

⁷ The Atkinson theorem says if the following three conditions are satisfied: a) the Lorenz Curve of distribution Y dominates the Lorenz Curve of distribution X; b) the distributions have an equal mean income or the Y mean is greater than the X mean income; c) the decision-maker is income-seeking and inequality-averse (i.e. the welfare function has positive first derivative and negative second derivative with respect to individual incomes); then welfare is higher in Y than in X.

⁸ $u'(c_i)$ decreases in c_i and is convex as long as the utility function $u(c_i)$ increases in c_i and is concave. All isoelastic utility functions considered in the note satisfy this condition.

V. CONCLUSION

The paper has reviewed the debate on discounting in the context of climate change policy, using two main frameworks: a “single world” approach, where people within and across generations are homogeneous (in two settings—deterministic and stochastic) and an “asymmetric world” approach, where differences in income distribution and preferences among people are taken into account.

In the context of a single world framework, without uncertainty, we have shown that the discount rate depends essentially on two factors, which can either drive the rate upwards or downwards, depending on the value judgments attached to these factors. The first relates to how much we value the welfare of future generations. If we value the welfare of future generations as much as the welfare of our generation—this component of the discount rate should essentially be very close to zero. Not everybody agrees, but the case from a human development perspective is very strong. The second factor is related with how much the welfare of people changes as their income changes. Is a one percent increase in income worth the same—proportionally—to a rich than a poor person? If so, this implies that this second component of the discount rate is low (close to one). Together with the first factor being zero, this implies a low discount rate. If, instead, we consider that increments in income should be worth less to the rich than the poor (perhaps because future generations are likely to be richer and are therefore likely to value an additional dollar less than our poorer generation today) then this second factor should be higher and in turn, the discount rate would go up.

However, choosing a discount rate based on these parameters is not the only limitation of this framework of analysis. In a deterministic single world scenario, the derivation of the discount rate does not take into account uncertainty in its application to climate change. If uncertainty is explicitly taken into account, this makes the social discount rate itself uncertain and thus a unique rate cannot be determined. Uncertainty might completely overwhelm any effect (and choices) we can make about the discount rates and the parameters that determine it.

The debate on discounting is extended further if instead of considering the world as a single and homogenous place we incorporate asymmetries such as differences in income distribution and preferences. These approaches show that the discount rate would be reduced—calling for urgent action on climate change. For example, if we use population-weighted growth rates and we simulate the future global growth rate with the threat of climate change, the potential growth rate will be lower and thus lead to a lower discount rate. If we consider heterogeneity in the rate of pure time preference, various authors have shown that the discount rate will be reduced. Gollier and Zeckhauser (2005) analyze the effect of such heterogeneity on aggregate discount rate. They conclude that the representative agent will have a declining discount rate when individuals have decreasing risk aversion preferences even though all members of a group have a constant discount rate. If we consider heterogeneity in the elasticity of marginal utility of consumption, several authors suggest that this would lower the discount rate. For example, Dasgupta (2007) argues that as climate change will have a disproportionate negative impact on the

world poor urgent action is needed to avoid the increase in inequality between the poor and the rich. Atkinson and Brandolini (2006) and Atkinson (1970) argue that the elasticity of marginal utility of consumption should not be constant and show why intra-generational inequality will reduce the discount rate.

Taking into account all these complexities in the determination of the discount rate, one can understand why it is not as simple to advice on climate change policy as suggested by many cost-benefit studies. Given that the world is not a homogeneous place and many of the issues surrounding the debate of climate change cannot be fully captured by the discount rate, relying on such a controversial and complex instrument might not be the best tool for analyzing the costs and benefits of climate change mitigation. Some authors claim that the uncertainties surrounding climate change are so deep that they overwhelm any other consideration. And the entire framework of comparing income across time using a discount rate collapses. As a result, arguments similar to those that justify insurance may provide a more relevant frame of analysis, where people pay today to avoid a potentially much larger, albeit uncertain, loss of income in the future.

APPENDIX: DERIVATION OF THE RAMSEY FORMULA FOR DISCOUNTING

We assume a single, infinitely-lived, representative household that maximizes utility, subject to an intertemporal budget constraint, as pioneered by Ramsey (1928) and thereafter developed by Cass (1965) and Koopmans (1965).⁹ This can also have an intergenerational interpretation, corresponding to the idea of a chain of finite-lived households connected through altruistic intergenerational transfers of income. The current generation maximizes dynastic utility and incorporates a budget constraint over an infinite horizon.

The model assumes that there is only one consumption good, one type of consumer and markets are perfectly competitive. In its more general form, total utility today is given by integrating from now into infinity the flow of per capita utility $u_t[c(t)]$ in each period t , where the consumption function varies with time:

$$U(0) = \int_0^{\infty} u_t[c(t)] e^{nt} dt$$

The term e^{nt} represents population growth, representing the additional flow of utility than results from the increase in the number of households. The dependency of the utility function on time accounts for the possibility that in each period the way in which utility is derived from consumption differs from the previous period and from the next.

A crucial simplification of the more general form of the utility function $u_t[c(t)]$ is to assume the following explicit form for the dependency of the utility function on time:

$$u_t[c(t)] = u[c(t)] e^{-\rho t}, \rho > 0$$

This implies that there is a “perennial” utility function and that the time dependency of the utility flow is discounted by the discounting factor $e^{-\rho t}$. An intermediate possibility between the more general and this most restrictive possibility is to consider that the discount rate is a time dependent function $\rho(t)$. A time-dependent discount rate creates some technical problems in dynamic analysis because the stationary property may be violated, which may create time inconsistent situations. But if the analysis is not dynamic, then there is no problem with a variable discount rate. In the rest of the note, we assume a constant discount rate.

Discounting with a positive rate, at the most fundamental level, tells us that what happens far into the future matters less than what happens closer to today for today’s decisions. This is why ρ is called the “rate of pure time preference of utility”. From an individual perspective, discounting implies that there is a preference for deriving utility today, rather than tomorrow, and it is a measure of our “impatience.” The higher the value of ρ , the

⁹ The derivation and discussion of this section draws in part on Barro and Sala-i-Martin (1995); Mas-Colell, Whinston and Green (1995); and Romer (2001).

more we discount the future, and the more we weight utility today. In intergenerational terms, the rate of pure time preference can be interpreted as how much value we attribute today to the utility that our descendants will derive. A low rate of pure time preference indicates that we put a high value on whatever level of utility they will derive from consumption. The higher the pure rate of time preference, the lower the value we give to the utility that people in the future will derive.

Under the simplifying assumptions of discounting with a constant rate of pure time preference, the utility to be maximized is given by:

$$U(0) = \int_0^{\infty} u[c(t)] \cdot e^{nt} \cdot e^{-\rho t} dt$$

If $u[c(t)]$ is increasing and concave, the use of the discount rate ensures that the integral converges. For some, this is the main reason for including discounting, and there is a long-running debate in economics on whether discounting is a technical convenience or a substantive issue with economic meaning—starting with Ramsey himself who wrote in 1928 that discounting was a weakness of imagination.

We are seeking the solution for the problem of maximizing utility subject to the intertemporal budget constraint. Household income derives from selling labor, at wage $w(t)$, and from remunerating capital assets $k(t)$ owned by the household, at $r(t)$:

$$income(t) = w(t) + r(t) \cdot k(t)$$

Income can either be consumed or saved—with the saved portion adding to the capital stock. Therefore, the per-capita budget flow equation, defining the budget constraint to which the household is subjected to is:

$$\dot{k}(t) = w(t) + r(t) \cdot k(t) - c(t) - n \cdot k(t)$$

meaning that capital assets per person increase with income, and decline with per-capita consumption and population growth (we assume a closed economy and no borrowing is allowed in this simple model).

To obtain the consumption function of the representative household, the utility function is maximized subject to the limits imposed by the budget constraint. The Jacobian is:

$$J = u(c) \cdot e^{-(\rho-n)t} + \nu \cdot [w + (r - n) \cdot k - c]$$

where ν is the Lagrangian multiplier. The first order conditions to obtain the maximum of $U(0)$ are:

$$\frac{\partial J}{\partial c} = 0 \Leftrightarrow v = u'(c) \cdot e^{-(\rho-n)t}$$

$$\dot{v} = -\frac{\partial J}{\partial k} \Leftrightarrow \dot{v} = -(r-n) \cdot v$$

The transversality condition is:

$$\lim_{t \rightarrow \infty} [v(t) \cdot k(t)] = 0$$

To obtain the differential equation that rules the behavior of consumption, we eliminate v from the first order conditions to obtain the following differential equation:

$$\frac{\dot{c}}{c} = (\rho - r(t)) \cdot \left[\frac{u'(c)}{u''(c) \cdot c} \right]$$

This is the most general form of the differential equation defining the behavior of households. We will have to make some further assumptions to get an explicit solution for $c(t)$, but just from looking at the equation fundamental insights emerge.

First, note that if $\rho = r$ then $\frac{\dot{c}}{c} = 0$, meaning that if the pure rate of time preference equals the interest rate, then the consumption pattern would not vary over time.

Second, deviations from this flat pattern depend on: the difference between ρ and r ; and, the term in square brackets. What is the term in square brackets? Note that the elasticity of marginal changes in utility, $u'(c)$, with respect to consumption, c , is:

$$\frac{\frac{du'(c)}{u'(c)}}{\frac{dc}{c}} = \frac{du'(c)}{dc} \cdot \frac{c}{u'(c)} = \frac{u''(c)c}{u'(c)}$$

Thus, the term in square brackets $\frac{u'(c)}{u''(c)c}$ can be interpreted as the reciprocal of elasticity of marginal changes in consumption with respect to consumption—which, given the concavity and monotonically increasing properties of the utility function, is always a negative term. Thus, if we define the function $\eta(t)$ as:

$$-\frac{u''(c)c}{u'(c)} = \eta(t) > 0$$

then we obtain the following simplified general solution:

$$\frac{\dot{c}}{c} = (r(t) - \rho) \frac{1}{\eta(t)}$$

To reach a steady state in which both $r(t)$ and $\frac{\dot{c}}{c}$ are constant, the function $\eta(t)$ must be a constant η . We turn next to the determination of the steady state.

The requirement that $\eta(t)$ must be a constant η implies that by solving the differential equation:

$$-\frac{u''(c)c}{u'(c)} = \eta$$

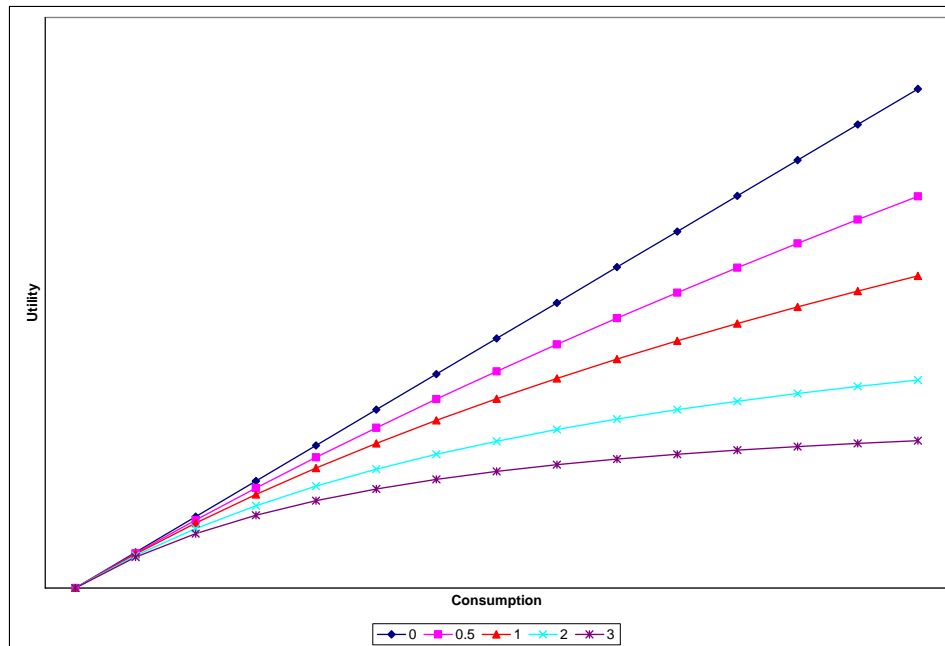
it is possible to get to the following explicit expression for the utility function¹⁰:

$$u(c) = \frac{c^{(1-\eta)} - 1}{(1-\eta)}$$

The constant η defines the curvature of the utility function, as illustrated in Figure 1. If η is (very close to) zero the utility function is (almost) linear in consumption and if η is one then we are in a log utility situation (taking the limit as $\eta \rightarrow 1$ we get simply $u(c) = \log c$).

¹⁰ Also known as a CES (Constant Elasticity of Substitution) or a CRRA (Constant Relative Risk Aversion) utility function.

Figure 1- Utility function for different values of η



A constant η also leads to the following simpler expression for the differential equation that rules consumption:

$$\frac{\dot{c}}{c} = \frac{1}{\eta} [r - \rho]$$

The elasticity of marginal utility with respect to consumption, η can be interpreted as the level of aversion to fluctuations in consumption, and thus can be seen as a measure of risk aversion. A high value of η implies that marginal utility is very responsive to changes in consumption (marginal utility falls very fast as consumption rises) and thus implies high risk aversion meaning that the individual is not willing to allow its consumption to vary over time. A low value implies the opposite and therefore higher risk tolerance—or higher tolerance for volatility in consumption. It can also be interpreted as the extent to which each individual allows for intertemporal substitution in consumption. It is possible, in fact, to prove that $\frac{1}{\eta}$ corresponds to the elasticity of intertemporal substitution of consumption. The proof is not presented, but the intuition is clear. If η is high, it means that marginal utility falls very rapidly as consumption increases over time, and so one percentage increase in consumption in today's dollars is much more valuable today than in the future. Thus, the individual is less willing to substitute consumption intertemporally (since η is high this implies that $\frac{1}{\eta}$ is low). A corollary of this interpretation is that η can also be understood as a measure of aversion

to interpersonal inequality within the same generation (intra-generational), because if we had chosen to index consumption instantaneously by individual, then the elasticity of the marginal utility with respect to consumption would still be given by η . If η is close to 1, this means that proportionate changes in consumption have the same proportionate impact on utility changes, regardless of how poor or rich different people are. If η is above 1, this implies higher aversion to interpersonal inequality, in the sense that a one percent increase in consumption is less valuable to a richer than to a poorer person. The coefficient η can be interpreted essentially in the same way in intergenerational terms.

The explicit solution for the consumption function over time is

$$c(t) = c(0)e^{\frac{1}{\eta}[r-\rho]t}$$

and when the utility function is logarithmic the consumption path is independent of η :

$$c(t) = c(0)e^{[r-\rho]t}$$

It is possible to derive an explicit expression for $c(0)$ based on the transversality condition, but we will stop here and focus on the dependency of the model on η . If $\eta < 1$ then the propensity to consume decreases as the average interest rate increases, meaning that the consumers are very responsive in reacting to intertemporal substitution. If $\eta > 1$ then the inverse effect occurs.

The results of the Ramsey model are also used to determine a rate of discount “ r ”. The differential equation that rules consumption when all the parameters are constant:

$$g = \frac{\dot{c}}{c} = \frac{1}{\eta}[r - \rho]$$

leading to the following expression as the Ramsey formula of the discount rate:

$$\boxed{r = \rho + \eta \cdot g}$$

We are “jumping” to the result that the growth of consumption per capita is constant, and equal to the growth of output per capita and of capital per capita, which is possible to show that in a Solow model would equal the growth rate of the labor-augmenting exogenous technological change. Without technological change, the growth rate would go to zero in a steady state.

The other important “jump” is to assume that the proper social discount rate is the interest rate as defined by the Ramsey equation—that is, private and social returns are the same, and we are assuming no external effects.

REFERENCES

- Arrow, Kenneth J. 1995. "Intergenerational Equality and the Rate of Discount in Long-Term Social Investment". IEA World Congress Paper, Stanford University. [<http://www-econ.stanford.edu/faculty/workp/swp97005.pdf>]
- Atkinson, Anthony. 1970. "On the Measurement of Inequality." *Journal of Economic Theory* 2: 244-263.
- Atkinson, Anthony and A. Brandolini. 2006. "Talk to Mirrlees 70 birthday seminar." July, London.
- Barro, Robert J and Xavier Sala-i-Martin. 1995. *Economic Growth*. Cambridge, Mass: MIT Press.
- Chiappori, Pierre-André and Christian Gollier, eds. 2006. *Competitive Failures in Insurance Markets: Theory and Policy Implications*. Cambridge, Mass: MIT Press
- Cline, William R. 1992. *The Economics of Global Warming*. Washington, D.C.: Institute for International Economics.
- Collier, Paul and Steve O'Connell. 2005. "Opportunities, Choices and Syndromes". Paper presented at AERC/Harvard workshop on Explaining African Economic Growth, Cambridge, Mass, March 18-19
- Dasgupta, Partha. 2007. "Comments on the Stern Review's Economics of Climate Change". University of Cambridge, Cambridge, UK. [www.econ.cam.ac.uk/faculty/dasgupta/STERN.pdf]
- Delong, Brad. 2006. "The Stern Review on Global Climate Change Once Again". The Semi-Daily Journal of Brad Delong, December 30, 2006. [http://delong.typepad.com/sdj/2006/12/the_stern_revie.html]
- Dietz, Simon. 2007. "Discounting the benefits of climate-change policy: the Stern Review, its critics, and policy implications". Paper presented at Envecon 2007: Applied Environmental Economics Conference, hosted by the UK Network of Environmental Economists (UKNEE), Royal Society, London, 23 March.
- Gollier, Christian 2002. "Discounting an Uncertain Future". *Journal of Public Economics* 85: 49-166.
- Gollier, Christian and Richard Zeckhauser. 2005. "Aggregation of Heterogeneous Time Preferences," *Journal of Political Economy* 113(4): 878-896.
- Harrod, Roy. 1948. *Towards a Dynamic Economics*. London: Macmillan
- Koopmans, Tjalling C. 1965. "On the Concept of Optimal Economic Growth". In *The Economic Approach to Development Planning*. Amsterdam: Elsevier.
- Li, Chuan-Zhong and Karl-Gustaf Lofgren. 2000. "Renewable Resources and Economic Sustainability: a Dynamic Analysis with Heterogeneous Time Preferences." *Journal of Environmental Economics and Management* 40: 236-250
- Mas-Colell, Andreu, Michael D. Whinston and Jerry Green. 1995. *Microeconomic Theory*. New York: Oxford University Press.
- Nordhaus, William D. 1994. *Managing the Global Commons: The Economics of Climate Change*. Cambridge, Mass: MIT Press.
- _____. 2006. *The "Stern Review" on the Economics of Climate Change*. NBER Working Paper 12741. National Bureau of Economic Research, Cambridge, Mass.
- Pigou, Arthur. 1932. *The Economics of Welfare, 4th edition*. London: Macmillan.

- Ramsey, Frank P. 1928. "A Mathematical Theory of Saving". *Economic Journal* 38 (December): 543-559.
- Romer, David. 2001. *Advanced Macroeconomics*. Second edition. New York: McGraw Hill.
- Solow, Robert. 1974. "The Economics of Resources or the Resources of Economics". *American Economic Review* 64(2):1-14
- Stern, Nicholas, S. Peters, V. Bakhshi, A. Bowen, C. Cameron, S. Catovsky, D. Crane, S. Cruickshank, S. Dietz, N. Edmonson, S.-L. Garbett, L. Hamid, G. Hoffman, D. Ingram, B. Jones, N. Patmore, H. Radcliffe, R. Sathiyarajah, M. Stock, C. Taylor, T. Vernon, H. Wanjie, and D. Zenghelis. 2006. *Stern Review: The Economics of Climate Change*. HM Treasury: London.
- Weitzman, Martin L. 1999. "Just Keep Discounting, But..." In Paul R. Portney and John Peter Weyant. *Discounting and Intergenerational Equity*. Washington, DC: Resources for the Future.
- _____. 2007. "The Stern Review of the Economics of Climate Change: Book Review for JEL". Harvard University, Cambridge, Mass.
[www.economics.harvard.edu/faculty/Weitzman/papers/JELSternReport.pdf]